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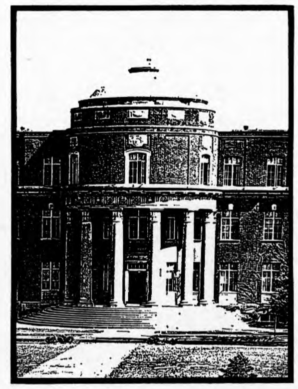

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Post-Yield Mechanics of Rock and Soil

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Introduction

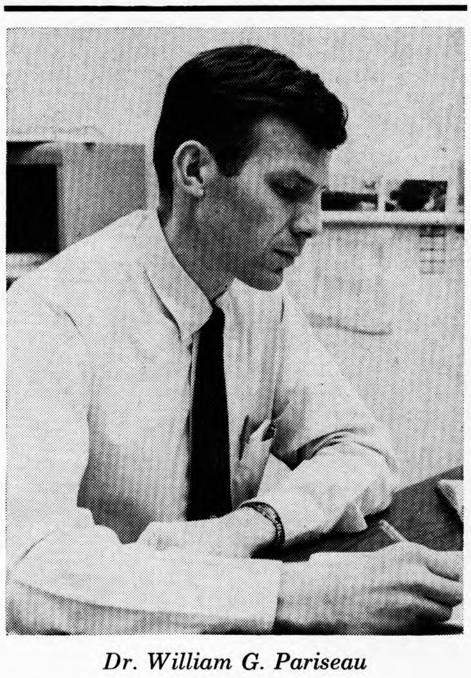
The general problem of improving mining operations through advances in technology resulting from research provides a common meeting ground for the engineer engaged in production and the research oriented engineer working mainly in the laboratory. Both areas of endeavor involve problems that deal with geologic materials and their response to loads imposed on them during drilling, blasting, loading, and haulage operations, and by gravity. According to Taylor,[48] a "soil" may be defined as a network of solid particles enclosing voids or interspaces of varying size. By this definition, sand, gravel, and broken rock are "soils."

Experimental Results

Our knowledge concerning the response of geologic materials to applied load is derived almost exclusively from laboratory studies under simulated environmental conditions. The effects of confining pressure, pore pressure, temperature, and strain rate on the deformation of rock and soil have all been investigated in various degrees of detail. The triaxial compression chamber is, by far, the most frequently used device in such studies. Most of our information about rock and soil behavior relates, therefore, to axially symmetric stress states.

Confining Pressure

The effect of confining pressure on strength is now well-documented. By strength, that is, yield strength, we mean the stress reached at or near the elastic limit of deformation. A linear stress-strain curve is usually considered indicative of elastic behavior. Extensive tests of rock properties carried out by the Bureau of Mines[52,53,3,4] show that rock at atmospheric pressure is generally elastic and fails by fracturing. Tests of rock conducted by the Bureau of Reclamation[1] to confining pressures of several thousand pounds per square inch (psi) show that strength is increased in proportion to the increase in confining pressure. Bredthauer[5] tested a variety of rocks to 15,000 psi and found that compressive strength continued to increase with confining pressure. The works of Robertson,[39] Handin and Hager,[21] and Patterson[36] representing tests on limestones, marbles, dolomites, sandstones, quartzites, shales, siltstones, slates, granite, and diabase show that the strength of jacketed specimens will increase with confining pressures to 60,000 psi. The renowned experiments of Bridgman[7] reveal the same trend to even higher pressures. The greatest enhancement of strength occurs at the lower confining pressures. As confining pressure continues to increase, the yield strength increases at a diminishing rate.



Dr. William G. Pariseau

Ultimate strength is the stress obtained at the highest point on the stress-strain curve, and the amount of permanent straining that occurs prior to rupture is a measure of ductility. What has been said of the effect of confining pressure on yield strength can be said generally of ultimate strength and ductility. Because of strain hardening, ultimate strength may undergo a several-fold increase with confining pressure, while the yield strength increases much less.

Pore Pressure

In compression tests on porous geologic materials, it is possible to control the chamber or confining pressure and the interstitial fluid pressure separately. The difference between the confining and pore pressures is known as the *effective* pressure. What has been said concerning the influence of confining pressure on the strength of dry or unsaturated jacketed specimens can be said of the influence of the effective pressure on the strength of porous materials. Two qualifications are required. First, the pore spaces must be interconnected, and second, movement of the pore fluid must be permitted. Interconnected pore spaces are necessary to insure a uniform distribution of pore pressure throughout the test specimen; free movement of the pore fluid transfers the load to the solid skeleton.

In soil mechanics, the change in volume resulting from the expulsion of pore fluid under compressive loads is known as consolidation. Materials of low permeability typically require long periods of time to consolidate. Porous materials can, therefore, be expected to exhibit time effects. For this reason, compression tests of clays and shales are not always easy to interpret. Nevertheless, tests performed on fully consolidated materials or as drained tests substantiate the concept of effective stress or pressure. Effective pressure is, in fact, an accepted principle in the domain of soil mechanics.

In rock mechanics, the works of Robinson,(40) Serdengecti and Boozer,(45) Handin(22) *et al*, and Schwartz(44) have demonstrated the validity of the concept of effective stress. An increase in effective confining pressure enhances yield strength, ultimate strength, and ductility. As an example, the ultimate strength of Berea sandstone increased from about 12,000 psi at zero effective confining pressure to over 60,000 psi at 29,000 psi effective confining pressure, roughly a five-fold increase in ultimate strength.(22)

Temperature

An increase in temperature lowers rock strength and increases ductility. Griggs(19) *et al* found in compression tests of dunite, pyroxenite, and granite nearly a three-fold decrease in strength as temperature was raised from 25° to 800°C. Basalt showed nearly an eight-fold decrease over the same temperature range. Dolomite showed a two-fold decrease, and marble an eight-fold decrease, in strength. Serdengecti and Boozer,(45) Heard,(25) Handin and Fairbairn,(20) and Handin(23) *et al* have also noted decreases in strength and increases in ductility with increasing temperature. The same trend continues at temperatures below room temperature. Heins and Friz(27) found that the modulus of rupture of a limestone, a basalt, and a granite were decreased 59 per cent, 49 per cent, and 19 per cent, respectively, at –320°F. The strain at fracture in the limestone was then approximately halved.

Strain Rate

Strain rate refers to the speed of deformation. Rocks and soils show higher strengths at higher strain rates. The effect of an increase in strain rate is more pronounced at elevated temperatures. Heard(26) found in tests on Yule marble that a million-fold decrease in strain rate (from 10^{-1} to 10^{-7}/sec) only slightly lowered the yield point stress at 25°C, but at 500°C the yield point was lowered by a factor of about 4. Serdengecti and Boozer(45) in tests on Berea sandstone and Solenhofen limestone also found that a decrease in strain rate decreased strength. Their published data show that ductility decreased with an increase in strain rate over a wide range of test conditions. An increasing strain rate has, therefore, an embrittling effect on rock. Note, however, that the strain to failure increases with an increase in strain rate. This is in keeping with the earlier results of Griggs,(18) who found that the shorter the duration of the test, the greater the deformation to failure.

The effect of strain rate on the strength of soil is quite similar to that on rock. Unsaturated sands are relatively insensitive to changes in strain rate over three orders of magnitude.(50) Saturated sands, though, may double in strength with a strain rate change from 10^{-3} to 10^{+1}/sec. Strain rate changes have a much greater effect on cohesive soils. According to Whitman(51) some clays become embrittled and fail by fracturing as strain rate is increased. In this case, strength may be doubled, whereas clays that tend to deform experience an increase of strength of the order of 50 per cent or less. As in the case of rocks, the greater the strain rate, the greater the strain to failure and the higher the strength. Thus, the effect of an increase in strain rate on the strength and ductility of geologic materials is to increase the former, decrease the latter and increase the strain to failure.

Brittle-Ductile Transition

Fracture at the limit of elastic deformation characterizes brittle behavior. Permanent straining vanishes, and the yield and ultimate strengths coincide. Ideal brittle behavior is, therefore, a limiting case of ductile response to load. Jaeger(30) has recently reviewed the subject of brittle fracture and experimental testing in rock mechanics.

Many rocks that appear brittle in unconfined compression tests become noticeably ductile at higher confining (effective) pressures and temperatures. Much of the violence of brittle fracture is, however, due to the testing machine.(10) Heard(25) specifically investigated the brittle-ductile transition of Solenhofen limestone as a function of temperature, confining pressure, and pore pressure. Gnirk and Cheatham(17) and Gnirk(16) found in penetration tests at room temperature and zero pore pressure that brittle action was suppressed with confining pressures as low as 500 psi (limestone). Sandstone, greenstone, and marble became ductile below 5,000 psi, while the transition pressure of dolomite was above 5,000 psi.

Whitman(51) has reported tests on clays in which a brittle behavior pattern was observed at atmospheric pressure and a ductile pattern seen under confining pressure. This suggests that soils having a relatively high cohesion may also undergo brittle-ductile transition.

Dilatancy

Another interesting and important facet of soil and rock behavior is dilatancy. Dilatancy refers to a relative increase in volume. Intuitively one would expect the volume of a test specimen to decrease under compressive loads. It is well known in soil mechanics, however, that packed sands expand when sheared. Loose sands decrease, and clays may increase or decrease in volume during shear.(29) Since stresses in soil mechanics seldom exceed 100 psi, volume change is synonymous with pore volume change. Rowe and Barden(42) have shown that contractive and dilatant zones may develop simultaneously under compression. Shockley and Ahlvin(46) showed this to be true even when gross volume changes were prevented. Roscoe(41) *et al* have shown that strain distribution is decidedly nonuniform, largely because of platen end friction, the influence of which, although noted nearly a century ago,(38) still does not seem to be fully appreciated (according to recent discussion).(8)

Rocks also exhibit dilatancy. Robinson(40) found that pore volume changed in a regular manner, decreasing with initial application of load, then increasing until the yield point was reached. At yield, pore fluid had to be continuously supplied in order to maintain test pressures which ranged to 10,000 psi. Handin(22) *et al*, em-

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ploying pressures to 29,500 psi, observed dilatancy whenever the ratio of pore pressure to confining pressure exceeded 0.8. Bridgman,[6] in tests on jacketed specimens of soapstone, marble, diabase, and several metals, showed by direct measurement that volume increases occurred under high compressive loads. Paulding,[37] in studies on granite, has also observed volume increase under high compressive loads.

Localized dilatancy under compressive loading has two sources, one in the shift of individual particle positions and the other in particle fracture. In nonporous rock, grain fracture must increase volume. Ultimately, dilatancy manifests itself as gross fracture of the test specimen. An adequate theory of failure must, therefore, take dilatancy into account.

The problem of platen end friction notwithstanding, compression testing will likely continue to be one of the principal tools in strength studies. Intuitively, direct compression appeals to us as having an obvious interpretation in terms of strength. At low pressures and room temperatures, it is a relatively simple test to implement. High pressure and temperature work is always time-consuming and expensive. Specimen preparation in either case is elaborate. Test specimen ends are usually finished to an optical flat in order to minimize the possibility of bending stresses arising during the test. Such care may not always seem warranted in view of the uncertainty introduced by end friction. Even then, suites of compressive strength data[12] on rock often have coefficients of variation over 35 per cent. Despite the vagaries of experimental testing, progress in understanding through research has been obtained, although often at great cost and with considerable frustration.

YIELD / CONST.	TRESCA (v. MISES) pl.str.	COULOMB	TORRE	GRIFFITH $2p \leqq q$	GRIFFITH $2p \geqq q$	MOD. GRIFFITH $p \leqq q$	MOD. GRIFFITH $p \geqq q$
n =	1	1	2	1	2	1	1
A=	O	$\frac{C_o+T_o}{C_o-T_o}$	$\frac{C_o+T_o}{2}$	1	$C_o/2$ or $-4T_o$	1	$\frac{C_o+2T_o}{C_o}$
B=	$C_o/2$ or $-T_o/2$	$\frac{-C_oT_o}{C_o-T_o}$	$\frac{-C_oT_o}{4}$	$-T_o$	O	$-T_o$	$-T_o$
sinø	O	A	$\frac{A}{2\|q\|}$	1	$\frac{A}{2\|q\|}$	1	A
form	line	line	parabola	line	parabola	line	line

Table showing the various yield criteria that are obtained by specialization of the constants n, A, and B in equation (1). ø is the angle of internal friction; C_o and T_o are the uniaxial compressive (+) and tensile (−) strengths respectively.

Theory

The objective in formulating a mechanical theory of behavior—indeed, of any theory—is the obtainment of a prediction capability. Without a theory, a chaotic state of trial and error will prevail; no design will be possible. A good theory, of course, should correspond closely to actuality.

The world of real materials is complex. Nonlinearity, anisotropy, heterogeneity, and irreversibility are the rules rather than the exceptions. Time and temperature effects are commonplace. A comprehensive material description over the widest range of conditions would, therefore, be quite unmanageable. As Drucker[13] points out, progress comes by making the simplest possible idealizations that are in keeping with the nature of the practical problem at hand. Once a problem is idealized, the idealization must be evaluated. Laboratory experimentation and field testing are the standard engineering methods for doing this. Naturally there is feedback in the process of constructing theory, idealizing a problem, and testing the resulting predictions.

In constructing a theory, we are not free to pull equations out of a hat. The conservation laws of mass and momentum, for example, must certainly be satisfied. In mechanics, these are expressed by the equations of equilibrium or motion. They do not depend upon the material; neither does the geometry of strain. What does depend upon the material is the stress-strain law or constitutive equation. A most familiar constitutive equation is Hooke's law, a linear, reversible, and time independent relationship between stress and small strains. It is the keystone of classical elasticity theory which has seen wide and successful application to many important technological problems.

A less familiar constitutive equation arises in plasticity theory. In plasticity theory, the material is assumed to behave elastically up to some limiting state of stress at which yield may occur. Plastic strain is, therefore, post-yield and permanent. Increments of strains are now assumed to be linear functions of the stress increments, but the form of the stress-strain relationship does not change with time.[49] However, the coefficients in the stress-strain relationship are no longer constant, and, hence, total strain may depend upon strain history.

The constitutive equation of visco-elasticity theory is an example of a stress-strain relationship which does depend on time, in contrast to elasticity and plasticity theory, in which the time variable does not appear explicitly. In an ordinary sense, though, strain is always time dependent; it never occurs instantaneously.

The application of a properly constructed theory will require two kinds of information. First, the rock or soil properties as indicated in the constitutive equation will have to be known. Properties of man-made materials such as steel and concrete are determined by standard laboratory tests. Rock and soil properties are also measured in laboratories, but the relationship between properties of laboratory sized specimens of geologic materials and field scale masses is not straightforward. In fact, there seems to be little correlation between the two.[31,32] It is a problem that could be most profitably researched.

The second kind of information needed in a theoretical analysis is a knowledge of the boundary conditions. Boundary conditions refer to the loads, displacements, velocities, etc., applied to the mass of material in question. With a knowledge of the applied "loads," we will be able through theory to predict what will happen throughout the entire body. This is why we wish to formulate boundary value problems, so that we can tell what is happening in the interior of a body from measurements on its surfaces which are accessible to us. For example, convergence measurements in stopes may permit the computation of stresses in the backs, and thus with a knowledge of the yield point stress indicate the proximity of failure. A great difficulty in stress analysis of underground excavations results from a lack of knowledge concerning the primitive or initial state of stress that existed prior to excavation; that is, the boundary conditions on an imaginary surface well away from the excavation are frequently in doubt. The primitive state of stress in the earth is a matter of continuing discussion.

Applications of Elastic Theory

All available experimental evidence has shown that elasticity theory reasonably describes the response of harder rocks such as basalt, granite, quartzite, and conglomerate to loads applied over time spans of engineering interest. It is applicable to soft rocks such as shales over a more restricted range of environmental conditions. In the case of soils, it serves as a basis for estimation in problems where the strains are linear functions of the stresses. Even when not strictly applicable, an elastic analysis is instructive since zones of possible stress concentrations are revealed along with the important geometric variables. An elastic stress analysis will nearly always be conservative, since any yielding that may occur tends to relieve stresses. It will, however, underestimate strain.

Problems in elasticity theory which have received extensive usage in mining engineering are the problem of a biaxially loaded plate containing a central hole and the problem of the long, thick-walled cylinder loaded laterally by internal and external pressures. The first corresponds to the "tunnel" problem and the second to the "shaft" problem. Simple beam and plate analyses have been made of roof conditions in deposits mined by room and pillar methods. Stresses about mine openings of shapes besides circular have also been analyzed. Most of these and other results are summarized by Caudle and Clark[9] and Obert[35] *et al.* More sophisticated elastic analyses of underground openings are those of Mindlin,[34] who solved the problem of a circular hole in a gravitating half space, and Berry[2] and Salamon,[43] who analyzed subsidence over longwall workings as an elastic phenomenon. The reflection theory of blasting is based on elastic wave propagation.[15,33] In soil mechanics, solutions to problems of loads applied to an infinite half space (the Boussinesq problem, for example) provide one means of estimating the bearing capacity of foundations.[49]

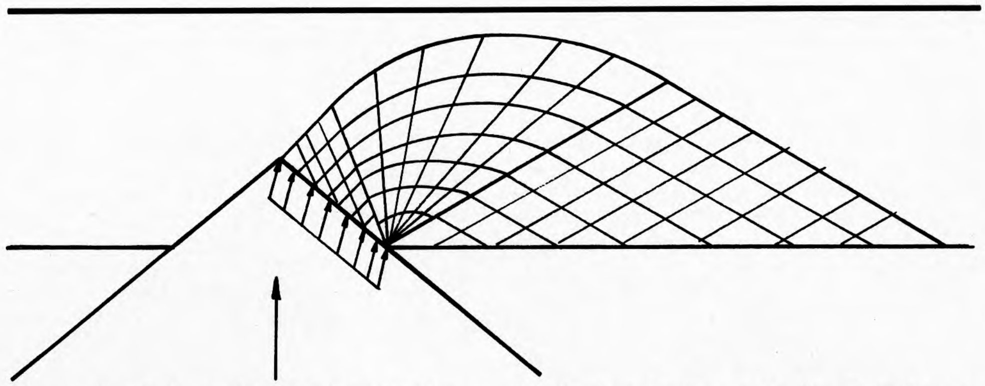

Fig. 1. A slipline field associated with a wedge having friction over its contact area penetrating rock or soil.

Applications of Plasticity Theory

Recognition of the fact that some yielding may occur without destroying the support capability of a body leads to a more realistic appraisal of its response to load. Elastic analysis indicates potential zones of stress concentration, but elastic design can be quite inefficient. The rather sizeable accumulation of experimental evidence shows that rock and soil yield and deform plastically over a wide range of environmental conditions. Even when a material is nominally brittle and fails violently in laboratory tests, fracturing from one boundary to another may not be possible in field size masses, and, hence, limit design will be reasonable. Fractures are often evident about the periphery of stable mine openings; many rock slopes, natural and man-made, are stable despite the presence of joints, fissures, and fractures. The range of situations found in mining which are amenable to plastic analysis extends over the entire spectrum of operations. As in applications of elastic theory, one analysis serves several practical situations. Scale has no theoretical standing.

Two-dimensional plastic problems have the property of being statically determinate in that a solution satisfying the equations of equilibrium and yield can be found without reference to a stress-strain law. Such solutions are incomplete and are described as solutions to problems of limiting equilibrium.

What might be called the general two-dimensional "punch" problem is the most frequently encountered case of limiting equilibrium. In this problem, a load is slowly applied to the surface of a large mass through the intermediary of a rigid "punch" or "die." A flat punch may correspond to a rigid foundation; a wedge-shaped punch may correspond to a chisel bit, bucket tooth, or the leading edge of a frontend loader; an inclined wedge might correspond to the cutting edge of an auger, drag bit, or coal plough. Blunt wedges and wedges with rounded noses and other shapes can be considered. If the punch is removed and in its stead load distributions are prescribed, we can then examine more general problems of foundation stability, and, with the introduction of inclined surfaces, problems of slope stability and retaining wall pressures as well.

Long pillars can be considered as material crushed between two opposing dies. Sequential and multiple penetrations can also be analyzed. The presence of friction over the "punch" face can be taken into account. In some cases where the weight of the material is negligible, analytic expressions for the limit loads can be obtained. Analytical expressions can also be obtained in a few instances where weight is not negligible. The computer can be made to handle the other cases. Sokolovski[47] presents the basic formulation and subsequent solution to many problems of limiting equilibrium of interest in soil mechanics. Harr[24] presents a blend of much of Sokolovski's work and that of other authors in his recent text on soil mechanics.

There are just three equations to be satisfied in any two-dimensional problem of limiting equilibrium. Two are equations of stress equilibrium, and a third expresses the condition of yield. The Mohr-Coulomb yield criterion is used almost exclusively in soil and rock mechanics; the Tresca and von Mises criteria are utilized in metal studies. Occasionally, the Torre, Griffith, and modified Griffith criteria are employed in rock mechanics. These are all compactly expressed by

(1) $|q|^n = Ap + B$ (Yield)

Where: q and p are one-half the difference and sum of the major and minor principal stresses respectively (compression is positive) and n, A, B are material constants.

The table on page 3 shows the values of n, A, and B for the various yield criteria. A and B are given in terms of the more familiar uniaxial compressive and tensile strengths, C_o and T_o. Since compression is positive, $C_o > O$ and $T_o < O$. The parameter ø is the angle of internal friction. The Griffith and modified Griffith criteria are branching formulas, that is, two statements are required to cover the entire stress range. A suggestion that arises from equation (1) and the table is that close analytical approximation to any experimentally determined yield function can be obtained by judicious use of a piece-wise continuous yield function. This would almost guarantee good agreement between theory and experiment.

By using the yield condition (1) in the equations of equilibrium, the system can be transformed into a set of ordinary differential equations. Thus, we have

(2) $dy = dx\tan(\theta \pm \mu)$ (Sliplines)

(3) $dq \pm 2q\tan\phi d\theta = O$ (Stresses along sliplines)

Where: θ is the angle from the x-axis to the major principal axis, $\mu = \pi/4 - \phi/2$, and the weight of the material is neglected.

If the weight of the material is important, then additional terms appear in equations (3).

Numerical forms for solution are readily obtained now by replacing the differentials by incrementals and the variables by averages. An iterative computational scheme follows. It is worth noting that if the sliplines as defined by equations (2) are known together with boundary values of stress, then the problem is solved. For this reason, the sliplines are of great assistance in solving problems of limiting equilibrium. In problems where the body forces are negligible, constant state regions where both sliplines are straight, and regions of radial shear where one family of sliplines are exponential spirals have analytical expressions, and consequently are much used.

Figure 1 shows a slipline field composed of constant state regions and regions of radial shear that are developed in the

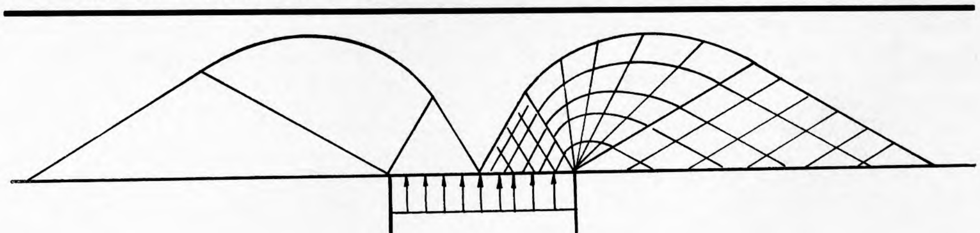

Fig. 2. The Hill type slipline field associated with a smooth, flat die indenting rock or soil.

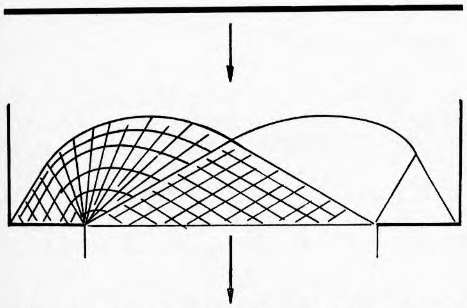

Fig 3. A possible slipline field associated with the flow of crushed stone or "soil" from a bunker having a wide slot outlet.

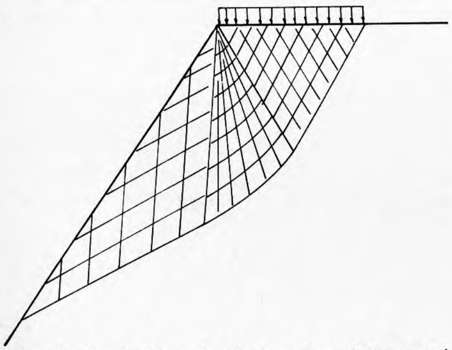

Fig. 4. A slipline field developed in a pit slope under a superincumbent distributed load.

analysis of a rigid wedge penetrating a Coulomb plastic material with friction along the wedge contact plane. Figure 2 shows a smooth, flat die also penetrating a Coulomb material. An interesting variant on the general "punch" problem arises in the analysis of gravity flow of broken material from bunkers having relatively large slot outlets. Figure 3 shows an approximate slipline field for this type of problem. Figure 4 shows a slipline field appropriate to a problem in slope stability.

A second large class of problems that arises from field situations and is amenable to plastic or post-yield analysis of a single geometry relates to shafts, tunnels, and boreholes. The same basic equations can be applied. Analytical expressions can be obtained by assuming that both families of sliplines are exponential spirals. Figure 5a shows a shaft or borehole with a yielded zone of Coulomb material surrounding it. Figure 5b corresponds to a borehole under internal pressure or to a blast hole under the action of gaseous explosion products. In Figure 6, gravity flow of a Coulomb plastic material from a wedge-shaped hopper is approximated. If the deformation about a tunnel is approximately radial, then the slipline field shown in Figure 5a can be applied.

Problems of limiting equilibrium in axial symmetry can be developed in much the same way as those of plane strain,[11] although the analysis is somewhat more involved and requires additional assumptions. One problem in axial symmetry of great interest is that of a cylinder crushed between two rigid dies. The cylinder might be thought of as a mine pillar or, perhaps, as a test specimen in the laboratory.

In a complete plastic analysis, the deformation as characterized by the strain-rate or velocity field must be displayed in addition to the stress field. Only if the velocity field is compatible with an acceptable stress field will a complete solution be obtained. Even then there is no guarantee of uniqueness unless the boundary to the plastic region is known. For this reason, solutions to limiting equilibrium, by themselves, are of doubtful value. Generally, the yielding process must be followed from its inception. This presents serious computational problems.

A difficulty that arises in velocity field considerations is that the plastic dilatation must always be expansive. Experimental evidence shows that this is true to a degree in both rock and soil, but whether either can expand sufficiently to meet the requirements of theory is doubtful. In many instances, incompressibility is a closer approximation to actual behavior. But for mathematical completeness, the velocity field described by

$$V = V_0 \exp\left[(\Theta - \Theta_0)\tan(\alpha \pm \mu)\right] \quad (4)$$

Where: α is the angle (constant) a streamline makes with a principal line

can be used in any of the slipline fields shown in Figures 1 through 6 by noting that Θ makes a constant angle with either the x-axis in rectangular coordinates (constant state region), or the r-ray in polar coordinates (regions of one or two exponential spirals). Equation (4) is a Coulomb plastic generalization of what Hill[28] refers to as streaming diagonal flow in metal plasticity.

One should also refer to the powerful limit theorems of Drucker and Prager[14] which may be used to estimate limit loads of bodies where detailed analysis is impossible.

Conclusion

On the basis of the experimental behavior of rock and soil, it appears that a large number of practical mining problems relating to drilling, blasting, materials handling, and ground stability can be analyzed as post-yield phenomena. Relatively solid rock as well as soils and sands yield according to similiar phenomenological criteria, the difference being mainly in the magnitude of the applied loads.

It has been said that the most practical thing in the world is a good theory. The large number of problems that are amenable to plastic or post-yield analysis would seem to indicate that much experimental verification and testing are warranted. In this respect, the fundamental studies in drilling of Cheatham and Gnirk,[16,17] are noteworthy. Their accumulation of experimental evidence substantiates the applicability of plastic analysis to wedge penetration problems in rock beyond the brittle-ductile transition. The need to extend stress analysis in rock and soil mechanics beyond the elastic range and the potential rewards for doing so are great and constitute a real challenge to the mining engineer.

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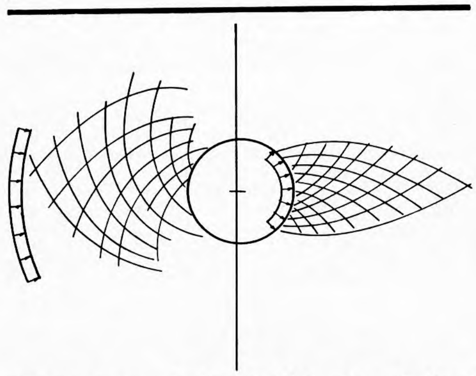

Fig. 5. (a) A slipline field developed about a borehole or shaft under external loading; (b) A slipline field resulting from a high internal pressure.

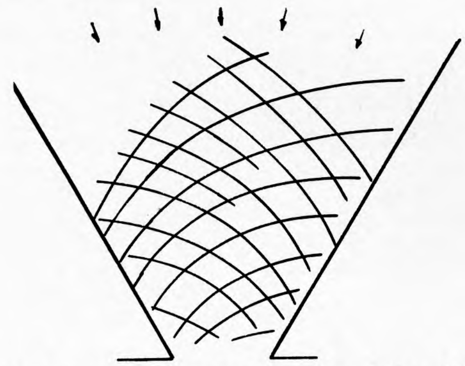

Fig. 6. A slipline field developed in a bin or hopper having smooth walls.

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Summer and Fall Meetings, Seminars, and Workshops

The following conferences and seminars are scheduled to be held at University Park under the general sponsorship of the College of Earth & Mineral Sciences during the coming summer and fall:

Ceramics Career Day

A "Career Day" designed to interest high school students and engineers from ceramics industries in ceramic science as a career will be held at the Conference Center on May 13, 1967, under the guidance of Professor F. A. Hummel, head of the Department of Ceramic Science, and Dr. Guy Rindone, professor of ceramic science.

22nd Annual Pennsylvania Ceramics Association Meeting

The 22nd Annual Pennsylvania Ceramics Association Meeting will be held at the Conference Center June 8-9, 1967, for the promotion of ceramic industry education in Pennsylvania. All college graduates affiliated with ceramic industries and management, production, and research are invited to attend. The annual review of the activities of the Department of Ceramic Science will be presented.

Combustion Engineering Seminar

A seminar on new developments in combustion engineering will be held at the Conference Center July 31-August 4, 1967, under the chairmanship of Dr. Peter H. Given, professor of fuel science and head of the department. The Program Director will be Dr. J. M. Beér, professor and head, Department of Fuel Technology and Chemical Engineering, University of Sheffield, Sheffield, England. The purpose of the seminar is to consider combustion processes in flames, heat transfer from flames to sinks in combustion, combustion aerodynamics, and partial modeling of flame processes. Other topics to be covered include electrically augmented flames, combustion driven oscellations, flame stabilization in high speed flow, spectroscopic methods for measurement of temperature, and concentration in plasmas.

This seminar is intended for engineers and scientists working in industry in research and development, design, or production.

Coal Industry Management

Under the direction of Professor R. B. Hewes, Department of Mining, the annual coal industry workshop, designed to provide instruction in good management practices in the coal industry, will be held August 20-25, 1967, at the Conference Center. This year the program content of the workshop has been revised extensively, and the lectures will be aimed specifically toward helping new or prospective section foremen. This is the first time that the workshop has been designed for a particular level of coal mining management. Speakers will include Paul S. Beaver, assistant director of Continuing Education; Charles W. Berry, research assistant, Department of Mining; Joseph Christoff, Jr., general superintendent, Pittsburgh Coal Company; Joseph W. Hunt, professor of mining engineering; Louis W. Lerda, assistant professor of continuing education; Robert E. Olson, manager, Industrial Engineering, Rochester & Pittsburgh Coal Company; Samuel Prichard, Jr., instructor of speech; and George H. Schenck, instructor in mineral economics.

Cooperative Program in Metallurgy

A meeting with representatives of business and industries who contribute to the Cooperative Program in Metallurgy will be held at the Conference center on September 8, 1967. It will be directed by Dr. Robert W. Lindsay, professor of metallurgy.

New Uses for Coal

A seminar on new uses for coal — the purpose of which is to review, for the benefit of research and development personnel in industrial organizations, the nature of coal, its preparation for use, and the problems of using coal in new processes, particularly in the production of industrial gases, liquid fuel, and chemicals — will be held at the Conference Center October 2-6, 1967, under the chairmanship of Dr. Peter H. Given, professor of fuel science and head of the department.

Inorganic Material Preparation and Characterization

The Materials Research Laboratory of the University's Institute for Science and Engineering will present a two-week course designed as an integrated presentation of the principles of solid state chemistry appropriate to predicting, synthesizing, and preparing in special form a desired or a new material, and its subsequent detailed characterization. This course, to be held at the Conference Center September 18-29, 1967, will be led by Professor Rustum Roy, director of the Laboratory and professor of geochemistry.

Post-Yield Mechanics of Rock and Soil

(Continued from page 5)

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About the Author

After serving three years in the Marine Corps, Dr. William G. Pariseau graduated magna cum laude in 1960 from the University of Washington with the B.S. degree in a geological engineering option. Awarded the Ph.D. degree at the University of Minnesota in mineral engineering, he subsequently joined the faculty of the Department of Mining at Penn State in 1966 as assistant professor of mining engineering.

Between sessions of formal education, he has held a variety of positions related to engineering. He is a member of Phi Beta Kappa and Tau Beta Pi. His major interests are rock mechanics and materials handling.

New Equipment for Mineral Constitution Laboratories

Over the past several months a number of important new items of equipment have been added to the Mineral Constitution Laboratories from funds made available by Dr. Osborn, vice president for research, the General State Authority, and other University sources. The four principal items are:

Perkin-Elmer Model 621 Grating Infrared Spectrophotometer

This instrument is a replacement for the Perkin-Elmer Model IR-21 instrument which has been in the Mineral Constitution Laboratories for many years. Among the advantages of the new instrument are the following:

(a) The independent variable, distance along the chart, is proportional to the wave number, covering the range 4000-200 cm^{-1} (2.5μ-50μ).

(b) The resolution is better than 0.3 cm^{-1}.

(c) Ordinate expansion of 5x, 10x, or 20x of any portion of the scale is available as well as a continuously variable expansion from 1/4x to 5x.

(d) The ordinate covers the range 0-100% transmission as well as 0-1 linear absorptance.

This instrument will be used in the analysis of organic compounds in solution and in the study of the vibrations of molecules or the strength of chemical bonds in solids.

Perkin-Elmer Model 303 Atomic Absorption Spectrophotometer

Absorption spectrophotometry is a recently developed technique for quantitative analysis by measuring the percentage absorption from a standard emission light source by the resonance lines characteristic of the elements present in the sample. This technique is particularly well suited for the precise determination of lead, zinc, cadmium, mercury, nickel and other elements which are difficult to determine by emission spectrometry due to their volatility. It is also frequently used for determination of sodium, potassium, calcium, magnesium, etc.

Our unit is set up with a recorder readout and at present is equipped with hollow cathode lamps to act as radiation sources for the determination of Ca, Zn, Mg, Ag, Fe, Sr, Ba, Mn, Sb, Pb, and Cd.

Applied Research Laboratory Model AMX Electron Probe

This is a replacement for the ARL Model EMX probe which has been moved to the Materials Research Laboratory for use in research on characterization of materials. The AMX probe was specifically designed for service work. Its performance is expected to be very similar to the EMX with the exception of greater serviceability and additional capability to handle samples in thin section form.

Siemens X-Ray Spectrometer Model SRS-1

This is a replacement for our present Norelco fluorescence unit, which was one of the very first fluorescent X-ray spectrometers to become available. Housed on the X-ray generator will be a Kratky Small Angle Scattering X-ray Camera which can be used for investigation of structures in glass, and for certain kinds of research on atomic binding in solids. The Siemens unit will have the following features not available on previous equipment:

(a) Very high intensities for the fluorescent X-ray determination of all elements of atomic weight greater than that of fluorine ($Z=9$).

(b) High wattage chromium and gold X-ray tubes.

(c) Four pre-aligned analyzing crystals.

(d) A pulse scope which displays the energy spectrum of the X-ray radiation and the discriminator or window.

(e) Special sine-function preamplifier coupled to two theta with first and second order selection switch.

(f) Constant potential X-ray generator capable of 4000 watts power.

It is the policy of the laboratory to provide the widest array and the most up-to-date instruments possible for analytical and morphological studies on rocks, minerals, and materials. The Mineral Constitution Laboratory's services and equipment are available for use by industrial organizations and other universities on a time-available basis, as well as by members of the faculty and students of The Pennsylvania State University. Those desiring services should write to Mr. Norman Suhr, 311 Mineral Sciences Bldg., University Park, Pa. 16802.

Fletcher L. Byrom Named McFarland Award Recipient

Fletcher L. Byrom, president of Koppers Company, Inc., has been named as the nineteenth recipient of the David Ford McFarland Award for Achievement in Metallurgy. This award, given annually by the Penn State Chapter of the American Society for Metals to an alumnus of the Department of Metallurgy at The Pennsylvania State University, recognizes outstanding achievement in some aspect of the metallurgical profession.

The award will be made at a banquet meeting to be held at 6:30 p.m., May 20, at the Centre Hills Country Club, State College, Pa. Following the banquet, Mr. Byrom will speak on "A New Stage For Steel — With a New Cast."

Mr. Byrom was graduated from Penn State in 1940 with the B.S. degree in metallurgy. Following two years with the American Steel and Wire Company as sales engineer and a three year stint during World War II with the Naval Ordnance Laboratory, he joined Koppers Company, Inc., as Assistant to the General Manager of the Tar Products Division. He became president and a member of the Board of Directors in 1960.

In addition to acting as chairman and board member of a number of companies and organizations, Mr. Byrom is a member of the Advisory Board of the McKeesport Campus of The Pennsylvania State University and of the Executive Committee, Alumni Council, Penn State Alumni Association. He is currently President of the Penn State Alumni Association.

Individual tickets for the banquet, which is informal, are $4.75, with tax and gratuity included, and may be secured by sending a postal money order or check for the number of tickets desired to Dr. George Simkovich, 122 Mineral Sciences Building, The Pennsylvania State University, University Park, Pa. 16802. Tickets will be mailed for orders received before May 10 but will be held at the door for monies received after that date.

Graduate Wins Award

Oded Rudawsky, a native of Israel, has been awarded a Doctoral Dissertation Fellowship in Natural Resources from Resources for the Future, a division of the Ford Foundation, for which the total stipend is $5,300 per year. Mr. Rudawsky's fellowship, one of ten awarded, was made as the result of a competition that attracted candidates from economics departments of universities all over the United States.

Mr. Rudawsky received the B.S. degree in mineral economics from Penn State in June 1965 and the M.S. degree in the same field in December 1966. His M.S. thesis was a study of the sources of supply of the fertilizer industry of Latin America, and the topic of his Ph.D. dissertation is "The Contribution of the Cement Industry to the Economy of Developing Nations." His work on his master's thesis was supported by a grant from the Occidental Agricultural Chemicals Corporation of New York.

Reprints Available

Recent publications of the College of Earth & Mineral Sciences are listed below. Those desiring reprints should address their requests to the author whose name appears in italics (if there is more than one), 5 Mineral Industries Building, University Park, Pa. 16802.

Title, Author, and Source

Reactions of Quartz and Corrundum with Aqueous Chloride and Hydroxide Solutions at High Temperatures and Pressures. G. M. Anderson and *C. Wayne Burnham*. Amer. Jour. Sci., 265, 12-27, Jan. 1967.

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Phase Relations in the System PbS-PbTe. M. S. Darrow, *W. B. White,* and R. Roy. Trans. of AIME, 654-658, 236, May 1966.

Hydrology of a Karst Area in East Central West Virginia. *W. B. White* and V. A. Schmidt. Water Resources Res., 2, 3, 1966.

DEIKE BUILDING DEDICATION

Formal dedication of the new Deike Building is scheduled for June 24, 1967, at 10 a.m. in the Earth & Mineral Sciences Auditorium, 26 Mineral Sciences Building. Participating in the ceremony will be John T. Ryan, Jr., president, Mine Safety Appliance Company; George H. Deike, Jr., vice president, Mine Safety Appliance Company; Dr. E. F. Osborn, vice president for research; Dr. C. L. Hosler, dean of the College of Earth & Mineral Sciences; and former deans of the College.

The program of events for that morning is as follows:

9 a.m.—Coffee Hour in the foyer of Deike Building

10 a.m.—Dedication Ceremony

11 a.m.—Guided tours of the new building.

Visitors will also be welcome to inspect the Deike Building on Friday and Saturday afternoons.

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