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LABORATORY STUDIES ON THE
GRINDABILITY OF ANTHRACITE
AND OTHER COALS

BY

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STATEMENT OF TRANSMITTAL

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SR-1 The Crushing of Anthracite May 31, 1958
SR-2 Petrographic Composition and Sulfur Content of a Column of Pittsburgh Seam Coal August 1, 1958
SR-3 The Thermal Decrepitation of Anthracite September 15, 1958
SR-4 The Crushing of Anthracite with a Jaw Crusher November 1, 1958
SR-5 Reactions of a Bituminous Coal with Sulfuric Acid February 1, 1959

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SUMMATION OF RESULTS

It is likely that the future will show increased development of reaction and combustion processes in which coal is used in finely divided form in fluid suspensions. Consequently the size distributions and surface areas of ground coals will be of importance, and this report describes the measurement of surface area shape factors which enable surface areas to be calculated from sieving measurements. Because of its low chemical reactivity and high resistance to grinding, anthracite has a special place in studies of grinding. Preliminary results indicate that anthracite of a given sieve size has about 30\% more external surface than the corresponding size of a bituminous coal.

It would appear that coals ground in the Standard Hardgrove ring and ball mill have an effective lower size limit of about 1 micron. Using this lower size limit the fresh external surface area produced on grinding is proportional to the amount of grinding over a wide range of revolutions of grinding. The Hardgrove index, however, is not an accurate guide to the increase in surface area.

This report gives the preliminary results of an attempt to give a mathematical description of the process of grinding in a mill. Such a description would be of great value in the optimum design of mills for a given duty, since it is possible that a mill for grinding anthracite would require different design parameters than one for a comparable duty grinding bituminous coals.
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I. INTRODUCTION

Grinding is an important process in such widely differing industries as the coal industry, paint industry, cement industry, and electric power industry, but although there is a wealth of information concerning the practical performance of the many types of grinding machines in use, studies of the fundamental basis of grinding processes have not made much progress until recent years. Not only are such studies important in the field of grinding machine design, but it is likely that investigation of grinding strengths will give valuable information on the chemical and physical properties of the materials studied. In particular, the shape and surface area of ground coal particles is of great importance in the following processes:

(a) the combustion of pulverized coal in central power stations,
(b) the gasification of pulverized coal,
(c) the manufacture of electrodes from low volatile coals,
(d) the blending of coal fines in the production of metallurgical coke,
(e) the briquetting of waste coal fines to produce saleable products,
(f) the production of low temperature coke and coal chemicals in fluidized bed processes,
(g) the firing of open cycle turbines with pulverized coal, and
(h) the hydrogenation and chemical processing of coal suspensions.

The handling, combustion, gasification and chemical processing of coals dispersed in gases or liquids are regions in which much work is yet to be
done, and the influences of particle reactive surface area, surface chemical properties, porosity, hydrodynamic behavior and resistance to thermal shock will depend, to a certain extent, on the size of the particles, and on the physical and chemical changes occurring when coal is ground to fine sizes. Other important factors will be the size and distribution of the mineral particles associated with coals. The stress laid on fundamental studies of grinding mechanisms and process can be gauged by the number of papers published in this field in recent years. An examination of B.C.U.R.A. Fuel Abstracts (1) from January, 1957 to August, 1958 reveals that 27 papers have been published detailing theoretical and fundamental studies on coal strength and grinding, of which 14 are by British authors, 5 by Russian or Satellite authors, and 3 by authors from the United States. As the initial work on coal in this field was performed by Hardgrove (2) in the U.S.A. in 1932, it would appear that the initial lead of the United States has not been consolidated.

Anthracite is of particular interest in grinding studies because of its hardness and resistance to grinding, and because its disadvantage (for many processes) of low reactivity can be overcome by using it in finely divided form.

II. OBJECTIVES

The objectives of this study are as follows:

(i) to investigate the surface area-to-volume shape factors for a range of coal types,

(ii) to determine the relation between sieve size distributions and envelope surface area distributions,
(iii) to investigate the change in size distribution and surface area with grinding for a standard Hardgrove test machine,
(iv) to investigate the change of coal strength with size of ground material,
(v) to formulate a mechanism of breakage of coal particles,
(vi) to determine the influence of mineral matter on the Hardgrove grindability index, and
(viii) to relate the resistance to grinding of coals with their chemical and physical structure.

This report, which is in the nature of an interim report, deals mainly with items (i), (ii), and (iii) above. The ultimate aim of the research is to determine the breakdown characteristics of given coal sizes and, hence, to construct formulae expressing size distribution of product in terms of the coal properties, initial size, and the type and duration of applied grinding forces.

III. REVIEW AND DISCUSSION OF PREVIOUS WORK

When a literature survey of the field of grinding was commenced it was soon realized that a satisfactory critical review of the many papers published would be very lengthy and would take a considerable time to prepare. Therefore, the following review is limited to papers relevant to objectives (i), (ii), and (iii) of this report and is in no sense a complete review of grinding or the fundamental aspects of grinding. A more complete review is in the process of preparation.

On the assumption that breakage processes occurred by shear, Rittinger (3) stated that the work required for size reduction is proportional to the fresh, broken surface produced. Kick (4), however, postulated that the work required is inversely proportional to the diameter of the
particles produced. Walker, Lewis, McAdams and Gilliland (5) give a
generalized equation in the form

\[ dE = -C \frac{dD}{D^n} \]  

(1)

where \( E \) is the energy required to reduce particles of size \( D \) by a differ-
tential size increment \( dD \), \( C \) is a constant, and \( n \) is a constant. This
equation gives Rittinger's Law when \( n = 2 \), and Kick's Law when \( n = 1 \);
but apart from this fact, the equation has no fundamental significance
and is probably not even empirically correct except under the special set
of conditions in which Rittinger's Law applies. Gross and Zimmerly (6)
measured the surface areas of various size fractions of crushed quartz by
the dissolution rate of the material in HF. They also measured the size
distributions produced when the quartz was crushed in a free falling
impactor. By the use of this machine, they were able to define the energy
imparted to the quartz on crushing. They found that the surface area to
volume shape factor, \( k \), defined by

\[ \text{(Area)} (\mu) = \frac{\text{Volume}}{k} \]  

(2)

where \( \mu \) is the sieve size of the particle, varied from 12 for quartz in
the range 200/270 U.S. mesh to 48 for material in the range 14/20 mesh.
The factor is 6 for a sphere or a cube. The high values of \( k \) were attri-
buted to internal surface area of cracks within the particles, there being
fewer cracks in the smaller highly broken material. For unbroken quartz
spheres, they determined \( k \) to be \( 8 \pm 0.24 \) for sieve fractions within the
range of 20 to 200 mesh, which indicated that there was a significant
internal surface even in unbroken particles. To test the hypothesis of
internal area, they attempted to measure the envelope external area of
crushed particles by electroplating them with silver. This technique gave shape factors of 11 to 14 instead of the previous values of 12 to 48. In the drop weight crushing tests, particles of size ranges 10/14 mesh to 35/48 mesh were crushed under varying falling loads. They found that the new surface area of quartz produced was in all cases proportional to the energy imparted to the quartz by the falling load and concluded that Rittinger's Law was correct. This work suffers from the disadvantage that any very fine material produced in the crushing process would be lost on handling, although it might represent a significant fraction of the total surface area produced.

Bond and Maxson (7) estimated surface areas of ground ores by measuring the weight versus sieve size distribution, extrapolating the distribution to a lower size limit of 0.8 micron and assuming a shape factor of 6. The lower limit of 0.8 micron seems to have been selected on the basis that on wet grinding the turbidity of the settled solution did not increase after the material had been crushed below 10 mesh. In crushing to 10 mesh, occluded colloidal material in the ore was released, and it was assumed that this material was about 0.8 micron in size. Clearly, any very fine ore formed by repeated grinding would be obscured by the fine, extraneous material liberated on coarse crushing; but this is no justification for neglecting the surface area of this fine ground product. Also the shape factor assumed was probably basically in error, and may not even have been constant over the size range used for area measurement. However, they found that for ten different ores ground in a ball mill with initial sizes in the range 28 mesh to 200 mesh, the fresh surface produced was proportional to the number of revolutions of the mill
and independent of the size of feed or rate of feed circulation. On the assumption that a fixed fraction of the work put into the mill is utilized for size reduction, they concluded that Rittinger's Law was correct. Using an impact crusher similar to that of Gross and Zimmerly, they also found that the surface area produced was proportional to the energy content of the crushing hammer. Assuming 100% efficient utilization of energy in the impactor, they found laboratory and industrial ball mills to be between 60 and 65% efficient in producing fresh surface when grinding ores. On grinding petroleum coke, however, they found a ball mill efficiency of only 16%, which they attributed to a lubricating action of the coke particles.

Gow, Cambell and Coghill (8) ground ores in ball mills of various sizes and at various rotation speeds and expressed the efficiency of grinding as proportional to the fresh surface produced per horse power. They found this efficiency to vary significantly with the speed of the mill; and, therefore, Rittinger's Law did not apply exactly to grinding in the mills.

Dean (9) used a relation between surface area of magnetite particles and their coercive force to measure the surface area of magnetite crushed in a drop weight impactor. He found that the fresh surface area produced was proportional to work input in crushing.

The pioneer work on the application of Rittinger's Law to coal was done by Hardgrove (2,10). He ground standard sized 16 x 30 mesh coal samples in a standard manner in a ring-ball mill and calculated a proportional surface area increase from the sieve analysis of the product. He assumed that the shape factor and coal density remained constant throughout the size range and that shape factors were the same for different coals. The integration of the size distribution to a proportional area was carried
out assuming that the minus 325 mesh size had a mean sieve size of 25 microns. Using these areas, he found the fresh surface produced to be proportional to the number of revolutions of the mill over a restricted range of revolutions. When large amounts of breakage had occurred, the increase in surface area on further grinding was less than that predicted by the increased number of revolutions; and Rittinger's Law did not hold. This he attributed to blanketing of the grinding by the fines produced. He defined the grindability of a coal in terms of the increase in surface area produced compared to that of the increase produced on grinding a standard coal the same number of revolutions, 60 revolutions being chosen as the standard condition. Hardgrove later found that there was an empirical relation between the grindability defined in this manner, and the percent by weight of coal, \( p \), passing a 200 mesh sieve, the relation being

\[
\text{Hardgrove Index} = 13 + 3.465 \cdot p. \tag{3}
\]

The assumption made concerning the area of fine material, the relatively inaccurate mathematical technique of integration of the size distribution, and the non-applicability of Rittinger's Law when large quantities of fines are present make the Hardgrove Index a very arbitrary index. Hardgrove himself showed that the index was a reliable guide to mill capacity only for restricted conditions of milling where grinding is coarse or where fine powder is immediately scavenged from the mill by air sweeping. In addition to his explanation of the effect of fines as blanketing the grinding, an alternative explanation is that the coal becomes progressively stronger as it becomes smaller because weaknesses in the coal are broken out; and as fines build up, the machine is required to grind progressively stronger material. Another explanation is that the surface area attributed to fines
in his calculations is probably seriously in error; and the greater the amount of fines present, the greater the overall error involved.

Romer (11) attempted to overcome some of the obvious objections to Hardgrove's work by measuring the surface areas of ground coals using the Lea and Nurse air permeability method. This method gives the hydrodynamic or envelope area of the particles. Romer found that the grindability indices calculated using the direct area measurements were considerably different from the Hardgrove indices, the surface areas being much higher than predicted for the products of coals of high Hardgrove indices. He then showed, though only for a limited range of conditions, that Rittinger's Law applied when the load on the mill or the number of revolutions of the mill were varied. Thus, the non-applicability of Rittinger's Law in Hardgrove's original work was ascribed to inaccurate surface area measurements. Objections still remain to the surface areas obtained by Romer. The permeability method of area measurement is known to be inaccurate for a sample of mixed sizes in which the largest to finest size ratio is greater than 3 (12), and Romer actually measured samples of size range from sub-microscopic (less than one micron) to 44 microns. Also a certain amount of very fine material is lost during the grinding and sieving operations, and this surface area is not included in the measured area. Evidence presented later suggests that this material is less than 3 microns in diameter and, therefore, would represent a significant area even if it were only .1% by weight of the total sample.

Skinner (13) pointed out that the hydrodynamic area as measured by permeability methods is of primary importance in the combustion of pulverized fuel and that it should be used to measure shape factors instead of
assuming a constant shape factor with size.

Work done at the Fuel Research Station of the Department of Scientific and Industrial Research in England (14) compared the size distributions of products for five coals (ranging from high volatile coking coal to semi-anthracite) ground in ball, ring-ball, disc, hammer, and peg mills. The mills were set so that the coal product was 80% through 200 mesh. It was found that the size distribution for different mills and different coals was in all cases very similar, and in particular the distribution of material below 125 microns (120 mesh) was almost identical. It might be concluded from this that the size distribution of the bulk of the product is nearly the same for all coals and mills when the mills are operated to a prescribed classification regimen; and, consequently, laboratory tests on a ring-ball mill of the Hardgrove type should give information of direct value in industrial practice. Where oversize particles are objectionable, as in the combustion of low volatile pulverized fuel, it would appear that this is more a problem of efficient classification and return of oversize for regrinding than a direct milling problem. No information was obtained on relative rates of throughput per horsepower of milling work for the different coals.

Fitton, Hughes and Hurley (15) showed that when various coals were ground in a ball mill and the material below 200 mesh sieved out at frequent intervals during grinding, the production of less than 200 mesh material was almost linearly proportional to the number of revolutions of grinding up to 80% of conversion to fines. As the minus 200 mesh material was removed almost as soon as being formed, the linearity of production against work done would indicate that the energy required to produce this...
material was nearly independent of the initial size (between 16 mesh and 200 mesh) of the ground material. If the ground product decreased in mean size in direct relation to the mean size of the material being ground, then, applying Rittinger's Law, it would be expected that the weight of minus 200 mesh material produced would rapidly decrease with increasing revolutions. It seems reasonable to suppose that, in the grinding, the greatest amount of work is expended in producing fine material and that the amount of this fine material does not depend strongly on the initial size, providing, of course, that the initial material is much greater in size than the fines.

When the minus 200 mesh material was allowed to accumulate in the mill, the rate of its production fell rapidly with increase in revolutions. This may be explained by noting that as the amount of fines in the mill builds up, the fines will be reground, leading to greater ultrafine surface area without the production of any more minus 200 mesh material.

Callcott (16, 17), on the basis of a considerable number of tests with the Hardgrove machine, concluded that the dimensions of the machine, the operating technique, and the sample preparation should be carefully standardized if results are to be comparable between laboratories. He also concluded that the size distribution of all British coals, when ground according to the standard test, could be expressed in the form

\[(\% \text{ under size } \mu =) \quad P = 14.9 \left( \frac{\mu}{1584} \right)^{n} \quad (4)\]

where \(\mu\) is the sieve size in microns and \(n\) varies from 1 for hard coals to 0.5 for soft coals. This relation held up to sizes of 300 to 500 microns. Calcott suggested using \(P\) as an index of grindability in preference to the Hardgrove Index (this was also suggested by Frisch and Holder (18)). He
did not believe that the work on surface area increase during grinding justified the use of any index except a simple index of breakage defined by p. The significance of the grindability index p may be stated in these terms: If a certain coal produces 10 per cent of material below 200 mesh in the standard Hardgrove test and another coal produces 20 per cent, then it is likely that on grinding in an industrial mill, the first coal will have approximately half the throughput of minus 200 mesh material obtained with the second.

Bennett and Brown (19) argue that proofs of Rittinger's Law for coal are of little significance because the area of coal cannot be determined unequivocally. Since coal contains a large amount of internal area in the form of micropores and macro-cracks, it is not possible to measure accurately the increase in total area (total area being measured by B.E.T. or heat of wetting methods) on grinding, as the increase represents the small difference between two large quantities. On the other hand, the increase in envelope surface area (measured by permeability methods) may not represent the fresh broken surface produced since grinding will expose the internal area of cracks in the coal.

Callcott (20) also states that the increase in surface on grinding has no more significance than a representation of mean particle size produced. He analyzes the problem of grinding in the following manner: Given different sized feeds into a grinding machine, what will be the size distributions of the products? Or if different machines operate with different size feeds, how much of the difference in products is due to the different feed sizes and how much is due to differences caused by the machines? The factors involved are the way coal of a certain size breaks
down on crushing (which may be termed the breakage mechanism of the coal),
the probability of a given size being selected for breakage (the selection
mechanism), and the probability of certain sizes being returned for regrind-
ing (the classification mechanism). He emphasizes the fact that the prod-
uct must be represented by a size distribution and cannot be represented
by one simple figure such as a mean diameter or a surface area.

It is clear that if the size distribution of product of different coals
ground in different mills is the same for some specified classification
dimension, then the comparative throughput of the mills will be determined
mainly by the hardness of the coal defined by some form of simple compara-
tive test such as the Hardgrove grindability test. It would appear that
this is a first approximation to the truth in many cases. However, im-
provements in grinding efficiency, mill design, and preparation of ground
coal of preordained characteristics will come about by the more difficult
but more precise analysis proposed by Calcott. Analysis of this kind
is difficult to carry out because the only data easily obtained are
the size distribution and production rate of the ground coal from a
given series of grinding steps. It is necessary to postulate a form of
mathematical function to represent the breakage mechanism, another to
represent the selection function and then to derive from these a size dis-
tribution for comparison with experimental values. The process is essen-
tially one of trial and error, and up to the present, the development of
predicted size distributions from postulated mechanisms has been hampered
by the mathematical difficulty of handling the statistical functions
involved. By assuming that each piece of a solid broke down to a
distribution of particles of fixed ratios of the initial size and that the probability of breakage of a particle of a given size was independent of the size of the particle, Epstein (21, 22) showed statistically that after a large number of breakage events, the product distribution should approach an asymptotic logarithmic-normal function. Coal from laboratory or industrial mills does not in general fit this type of function (23, 24, 25), which is not surprising since the assumptions made are unlikely to be true. Also the fine product is usually removed from the mill before a large number of breakage events have occurred. Callcott gives a mathematical method of dealing with the functions involved in repeated breakage which offers great promise and which should be applicable in cases where assumptions other than those made by Epstein are used.

As presented above, a disadvantage of Callcott's method of approach is that it does not directly define the relation between size of product and work expended in grinding, either for a particular mill and different coals or between mills. This relation enters principally in the probability-of-selection function and the number of breakage events occurring, in the following manner. In many types of mill, the grinding process is actually a crushing operation performed on a fraction of the material present. Assuming that different coals have the same breakage mechanism, a weak coal can be considered to undergo more breakage events per crushing operation than a strong coal. Therefore, the probability of selection for additional crushing of the initial breakage-products is increased. Again, for different operation conditions (e.g. speed of revolution, ball volume to total volume, charge volume to total volume, weight of or force
applied to crushing elements, etc.) this selectivity function will be different. Thus, the definition of the amount of product and product size from a given feed size will be in terms of strength of coal and an analysis of the forces applied in any particular mill operating at given conditions; and this is implicit in Callcott's approach. A complete definition of amount and size of product in terms of the design and operation of a mill and strength of feed material is the perfect and complete solution of the milling problem. However, it is going to be extremely difficult to determine the selectivity function as a function of all the variables involved without making a good many assumptions or empirical measurements. For instance, it will not be correct to assume that the number of breakage events occurring in a single crushing operation is independent of the size of material crushed. As particles become smaller and smaller, a corresponding number of breakage events will produce greater and greater fresh surface, requiring more and more energy to break chemical bonds. Therefore, if the crushing operation puts in a fixed amount of work, as when a ball falls into the charge (ball and tube mills), obviously the number of breakages must decrease. Again, in a process where the particles are stressed by the friction and weight of a surface sliding over them, then as major faults in the structure are broken out by size reduction, the finer material will have to be stressed to a greater degree before it fractures. In effect, the surface has to slide further before a breakage occurs, and the number of breakages per revolution will decrease. In this particular example, the small material on the point of fracturing has a high energy content per unit volume; and on fracture, part of this
energy will be utilized in breaking chemical bonds and part will be emitted as heat of fracture. If Rittinger's Law applies, the fresh surface produced will be proportionally more than in a particle which fractures at lower stress. This implies that the breakage mechanism (which is the size distribution produced per breakage event) is not the same for different materials. If Rittinger's Law does not apply, the heat of fracture and heat loss by frictional slip over non-fracturing particles becomes greater for finer sizes; and the work input per breakage event increases. If Rittinger's Law applies, it will be possible to relate the breakage mechanism to the number of breakage events (selection mechanism), which will assist in allowing for unknown variations with particle size. Therefore, in spite of the objections raised as to the validity of Rittinger's Law, the authors believe that further examination of the Law is justified.

Rittinger's Law cannot in general be true, which can be seen if an extreme case is considered as follows. Let a grinding machine be grinding particles which have such strength that the forces imparted by the machine do not exceed this strength. Obviously work will be done without the production of fresh surface. Again, as a distribution of particles approach this strength, the particles of greater strength (present by probability) will either deform without fracture and release heat on release of the applied forces; or the applied forces will slip over the particle surfaces releasing heat as friction. However, if the forces in general are great enough to cause fracture and if the energy released by heat of fracture and friction is a constant fraction of the applied energy, Rittinger's Law might hold over quite a wide range of conditions. If this
were true, an experimentally determined breakage mechanism at a given size could be used to predict the number of breakage events occurring for a crushing operation in which the energy input was constant. Laboratory investigations will provide information on the mechanism of breakage, the validity of Rittinger's Law, and the effect of coal type on the selection function. The effect of operating conditions and mill design on the selection mechanism must be determined on industrial equipment, as must the classification mechanism.

Aries (26) has shown that the ratio of volume to surface area is a characteristic dimension for use in studies of the chemical reactions of solids. Hawksley (27) considers that the drag diameter, defined as the diameter of the sphere having the same surface area as the particle, is the most fundamental size for use in hydrodynamic problems involving particles. It can be shown that the drag diameter is related to the Stoke's diameter by

\[
d_d = \frac{k d_s^2}{6\mu}
\]

where \(d_d\) is the drag diameter, \(d_s\) the Stoke's diameter, and \(k\) and \(\mu\) the shape factors and sieve size of the particle as defined before. The above two examples show that the surface area to volume shape factor, \(k\), is of interest quite apart from any considerations of energy used to produce new surface.
IV. EXPERIMENTAL PROCEDURES

A. Characteristics of Coals Used in Tests.

Four coals were used, ranging from an anthracite of 4.5% volatile matter to a low rank bituminous coal of 42.5% volatile matter and 6.2% oxygen content. The analyses were performed on the 16 x 30 mesh coal which was the starting material for all tests, and the proximate and ultimate analyses were carried out according to A.S.T.M. standard procedures.

B. Preparation of Coal Samples.

Since the interest was in the properties of the coal actually used and not in those of the bulk sample supplied, no studies were made on differences in character between the bulk sample and the final sample. The sample for use was prepared from minus 1/2" material by passing it through a jaw-crusher followed by a disc mill and sieving out the 16 x 30 mesh fraction on a Rotap sieving machine. The 16 x 30 mesh fraction was removed after each pass through the jaw crusher or disc mill. Microscopic examination showed the sample to be almost free from adhering fines or agglomerates. Before use, the coals were spread on trays in a thermostatically controlled (to ± 1°F.) laboratory and allowed to reach equilibrium with the atmosphere. Grinding and weighing were performed in this laboratory.

C. Grinding of Coal.

The coals were ground in a standard Hardgrove test machine according to the A.S.T.M. standard method (28), both to measure the grindability and to provide sufficient fractions of material for surface area measurement. The coals were also ground for varying revolutions of the machine, ranging
from 3 revolutions to 140 revolutions. In every test, 50 gm. of coal were charged to the machine and the product sieved as described below.

D. Sieving of the Ground Coal.

It was considered essential that good performance of sieving was obtained; and consequently, a standard procedure was carefully followed in each case. The material from the mill was carefully brushed out into the top sieve of a series of 6 sieves\(^1\) (16 mesh to 120 mesh). The sieves were shaken in a Rotap sieving machine for 10 minutes, the material through 120 mesh removed, the sieve cleaned if necessary, and the sieves reshaken for five minutes. This was repeated for five minute intervals until the amount of minus 120 mesh material coming through was small. (A total sieving time of 25 minutes was always sufficient). The same procedure was then followed using the minus 120 mesh material in another series of 6 sieves (120 to 325 mesh), but the sieves required cleaning more frequently and a maximum sieving time of 35 minutes was sometimes required. Cleaning was carried out by separating the sieves a small amount, inserting a brush and brushing the underside of the top screen. The collected sieve fractions were weighed to the nearest 0.01 gm.

It was found that this series of multiple sievings gave weight losses outside of the tolerance given in the A.S.T.M. standard. Therefore, for standard Hardgrove Index determinations the ground coal was sieved through a 200 mesh sieve only and the multiple sieving performed after the initial weighings. It was found that when this was done the weight loss in the single sieving operation was within tolerance; and if the weight loss in

\(^1\)All sieve numbers refer to U.S. standard mesh.
the multiple sieving was assumed to be of material below 200 mesh, the
Hardgrove Index thus calculated was the same as that for the single sieving,
within the tolerance allowable (± 2%).

When the coal was ground for a few revolutions only and the amount of
fine material formed was small, the minus 120 mesh material was sieved
through tared 3 inch diameter sieves. Then the coal plus sieve was
weighed directly to the nearest 5 milligrams.

E. Moisture Loss During Grinding.

It is always assumed that the weight loss occurring on grinding is
loss of minus 200 mesh material. To test whether some of this loss was
moisture, even though the coal was in equilibrium with the atmosphere,
samples of dried coal weighing about 3 gm. were ground in a small flat disc
grinder to about the same size distribution as they would have had if ground
in the Hardgrove test. The moisture content of the coal was determined (by
drying at 105°C, to constant weight in an electric oven) before grinding
and after grinding and sieving.

F. Apparent Densities.

The apparent or geometric density of coals of different size fractions
is required in the determination of the hydrodynamic area. Densities were
determined at first by water displacement in a vacuum case specific gravity
bottle. The coal powder was weighed, vigorously stirred with water to
thoroughly wet and release entrapped air, and the density determined. The
operation could be carried out in a few minutes, and it was thought that
in this time the water would not penetrate the internal porosity of the
coal to any marked extent. This method gave reproducible results for the
larger mesh fractions, but it was found difficult to wet and de-aerate the fine mesh fractions (minus 200 mesh). It was therefore decided to determine mercury densities. The mercury densities were determined using a mercury porosimeter (29). In the finer fractions of material, the mercury had to be forced into the spaces between the particles. Therefore the pressure of mercury was increased until the rate of entry of mercury dropped suddenly and further pressure caused only a small further penetration of mercury. The sudden change point was taken as equivalent to the geometric density, and the slow additional penetration as the filling of the fine internal pore system of the particles.

G. Measurement of Surface Area of Coal Fractions.

The surface areas of various coal fractions, sieved from the ground product of a standard Hardgrove test, were determined using the liquid permeability method described by Lakhanpal, Anand and Puri (30). The apparatus is shown in Figures 1 and 2. A coal sample is weighed and packed into tube A, being supported by a thin pad of glass wool over the constriction in the tube. The tube is mounted on the apparatus as shown. With water through the bed and with stopcock B closed, the mercury is lowered in tube C to give a pressure difference across the bed. This pressure difference is noted in terms of the difference in mercury levels, $l_1$ (see Figure 1), plus the mercury equivalent of the water, $\Delta l_1$. With stopcock D closed, stopcock B is opened and the time taken for the mercury to rise in tube C is noted. Stopcock B is shut as the stopwatch is stopped, and the new value of pressure difference, $l_2$, is noted. The temperature of the water is noted and its viscosity determined from tables. The mean diameters of tube A and tube C were measured by mercury calibration. From the weight and density of the coal, the diameter of the tube containing
the bed, and the length of the bed, the bed porosity can be calculated. The surface area of the coal in cm.² per gm. of coal can then be calculated, as described in Section V A.

The method was found to be simple and quick and to give good reproducibility. The major difficulty was found in obtaining a bed free from air bubbles. When air was present, the bed had a characteristic mottled appearance at the surface of the tube; and surface areas were both too high and reproduced poorly. This difficulty was overcome by allowing the coal sample to soak in boiled-out water for an hour before use, with frequent stirring, and by packing the bed wet. If the coal was well wetted, and if it was packed under suction (from a water pump) with a continuous flow of boiled-out water, then air bubbles were not found in the bed. The bed was kept completely full of air-free water at all times during the testing.

The instrument was tested by measuring the surface area of a sample of glass spheres of size 100-200 microns, the glass having a density of 2.50 gm./cc. A microscope size count was made on the spheres, and the surface area calculated as described in Section V A (ii). Comparing the two areas gave a factor of $k_oq^2$ in equation (7) of $4.75 \pm 6\%$. Carman (31) reviews values of $k_oq^2$ found for spheres and quotes values from 4.5 to 5.1, with a best value of 4.8. Clearly our value is in good agreement with this, and it may be concluded that the apparatus is working satisfactorily.

H. Measurement of Sub-Sieve Size Ranges.

(i) Preparation of slides. The samples for microscope viewing were prepared in the following manner. A quantity of the material to be
examined was stirred vigorously in several ml. of toluene until well dispersed, and a drop of the suspension transferred to 1 ml. of a 10% ethyl cellulose in toluene solution. After stirring, a drop of this suspension was spread on a slide and allowed to thicken. Using this technique, extremely uniform and well dispersed particle fields were obtained, even for very fine particles. If the starting sample was a pad of dried settled material from a sedimentation, the pad was shaken in a tube containing toluene and leadshot and the resultant coal in toluene suspension used to prepare the slide.

(ii) Ratio of microscope diameter to sieve size. The characteristic microscope dimension chosen for microscopic sizing was the diameter, \( d_p \), of the circle having the same area as the projected area of the particle in the plane of the slide. This dimension is not in general the same as the sieve size of the particle. To obtain the ratio \( f \) defined by

\[
 f = \frac{d_p}{d}
\]  

microscopic size counts were performed on three sieve fractions, 325 x 230, 230 x 200, and 200 x 170, for each of the coals tested. The measurements were made by projecting the field on to a ground glass screen on which circles of varying radii were drawn; the field was then moved to bring each particle under the appropriate diameter circle. For the 325 x 230 mesh fractions, the measured particles were assigned to one of the following \( d_p \) groups: greater than 24 microns, 24-36 microns, 36-48 microns, 48-60 microns, 60-80 microns, 80-100 microns, 100-120 microns, 120-140 microns, 140-160 microns. From such a count, the cumulative number of particles below any given \( d_p \) was obtained. From this number, the size distribution,
and the specific surface area of the coal fraction tested (determined as described before), the factor f was calculated as described in section V B. Using appropriate size groups, similar counts were performed on the other sieve fractions. It was found that the sieve fractions tested included some fine particles ranging down to 1 micron in size. This fine material was clearly derived from dust adhering to the larger, true sieve sizes. Although the numbers of these fine particles were appreciable, it was calculated that their weight and surface area were negligible compared to the bulk of the material present. The lowest size fraction measured was chosen so that the area and volume of this fraction was negligible compared to the larger material and the assumption made that finer material would also be negligible.

With measurements of from 250 to 350 particles, the factor f could be duplicated within ± 5%. Three tests were performed on each sample, and the mean of the three results should be accurate within about ± 3%.

(iii) Size distributions of sub-sieve fractions of ground coal. To extend the cumulative weight versus sieve size to sub-sieve size particles, the sub-sieve fraction (minus 325 mesh) was sedimented into a fine and a coarse fraction and microscope sizing carried out on each fraction. Splitting into two fractions was necessary because it was impossible to measure the finest material \( (d_p \text{ minus } 1 \text{ micron}) \) and the coarsest \( (d_p \text{ about } 80 \text{ microns}) \) with one magnification system. About 0.5 gm. of the minus 325 mesh fraction was accurately weighed and stirred vigorously in distilled water in a 400 ml. beaker with about 0.1 gm. of wetting agent (Alconox). When the coal had been well wetted by standing with repeated stirring for
12 hours, the mixture was well stirred and allowed to settle for 1/2 hour. The supernatant suspension was carefully decanted, the sediment stirred with a fresh volume of water and wetting agent, and again allowed to settle for 1/2 hour. This operation was repeated until the settled liquid was fairly clear and again repeated, with .3 gms. of dispersing agent (sodium hexametaphosphate) instead of Alconox, until the supernatant liquid was almost completely clear at the end of the half hour settling period. The accumulated liquor and "floats" was evaporated to 100 ml., filtered through a tared gooch crucible fitted with filter paper, well washed, dried, and weighed. The collected "floats" were dispersed in toluene and slides prepared as described in IV H (i). The sediment, or "sinks", which was now free from any appreciable quantity of fine material was washed and slides prepared.Microscopic size counts were performed on each fraction using a magnification of x 100 on the "sinks" and x 600 on the "floats". The sink material was mainly from 12 to 80 microns in size, and the float material ranged in size from less than 0.8 micron to about 20 microns. A cumulative weight against size distribution was calculated (see V C) for each fraction; and since the respective weight of each fraction was known, a combined distribution could be calculated. The factor f determined previously was assumed to hold for the sub-sieve fractions, and thus microscope diameters could be converted to equivalent sieve sizes.
A. Calculation of Area to Volume Shape Factors.

(i) Specific surface area of tested material. The specific surface area of the material tested in the permeability apparatus was calculated from the following formulae

\[
d = \left( \frac{k_0 q^2}{2} \right) \left( \frac{R_i}{R} \right) \left( \frac{1 - \varepsilon}{\varepsilon} \right) \sqrt{\frac{\ln L \log \left( \frac{2}{L} \right)}{\varepsilon t}} \times 10^5 \tag{7}
\]

\[
S_o = \left( \frac{6}{d} \right) 10^4 \tag{8}
\]

which are derived in Appendix I. In equations (7) and (8), d is the surface area mean spherical diameter of the material in microns, \( S_o \) is the specific surface area in sq. cm. per cm\(^3\) of material, \( k_0 q^2 \) is a factor which varies with the shape of the pores in the bed, and the other terms are constants of the apparatus or experimental data as defined in Appendix I. The factor \( k_0 q^2 \) is 4 for circular pores, 4.8 for a bed composed of spheres and 5 for a bed composed of irregular particles (31). It should be noted that d is only an intermediate step in the calculation of the surface area \( S_o \), and it is not necessary to attach any particular physical significance to it. However, values of d were calculated because these values could be compared with the nominal sieve sizes of the material tested. These values, consequently, gave a ready indication of an unsatisfactory test. \( S_o \) is in no sense a mean of determined dimensions but is a direct measure of surface area. For the irregular particles of ground coal equation (7) becomes
(ii) Accuracy of permeability apparatus. In order to test the accuracy of the apparatus, the specific surface area of a sample of glass spheres was determined by microscope measurement, as described in V H, and by the permeability method as described in IV G. The specific surface area by microscope measurement was calculated by the following equation

\[ S_o = \frac{\int_0^\infty \pi \mu_i^2 dN_i}{\int_0^\infty \frac{\pi}{6} \mu_i^3 dN_i} \]  

(9)

where \( dN_i \) is the number fraction of spheres in the size range \( \mu_i + d\mu_i \). The integrations were performed graphically by plotting the cumulative number of particles below size \( \mu_i \) against \( \mu_i^2 \) and against \( \mu_i^3 \) and finding the appropriate areas under the curves. Combining equations (7) (8) and (9)

\[ \left( \frac{k_0 q^2}{4} \right)^{1/2} = \frac{d \cdot 10^4}{\sqrt{\frac{\int_0^\infty \mu_i^2 dN_i}{\int_0^\infty \mu_i^3 dN_i}}} \left( \frac{R_l}{R} \right) \left( \frac{1-e}{e} \right) \sqrt{\frac{\eta L \log \left( \frac{l}{d} \right)}{e t}} \]  

(10)

The mean value of \( k_0 q^2 \) determined from equation 10 was 4.75, which is in good agreement with the expected value.

(iii) Calculation of instantaneous area-to-volume shape factors. The specific surface areas per cm\(^3\), \( S_o \), of 10 sieve fractions of a coal ground according to the standard Hardgrove test (60 grinding revolutions) were
determined. As the weight of coal in any fraction, $\Delta p$ say, was known and the density $\rho$, was known, then the surface area $\Delta S$ of the fraction was calculated from

$$\Delta S = \frac{S_0}{\rho} \Delta p$$

(11)

The cumulative total surface area, $S$, and cumulative weight $p$ was then calculated from

$$S = \sum \frac{S_0}{\rho} \Delta p$$

$$p = \sum \Delta p$$

(12)

When $S$ was plotted against $p$ on semi-log paper, it was found that a smooth shallow curve was obtained. Then

$$\frac{dS}{dp} = \frac{dS}{d(\log p)} \left( \frac{1}{2.3p} \right)$$

(13)

and values of $\frac{dS}{dp}$ were obtained from the slope of the curve at any given value of $p$. The instantaneous specific surface area per gram at a sieve size $\mu$ corresponding to weight $p$ is clearly given by

$$S_\mu = \left( \frac{dS}{dp} \right)_\mu \text{ cm}^2 \text{ gm}^{-1}$$

(14)

If $\rho_\mu$ is the density of coal of size $\mu$ and the shape factor for this size is $k_\mu$, from equations (14) and (2)

$$k_\mu = \left( \frac{dS}{dp} \right)_\mu \rho_\mu$$

(15)
It should be noted that \( k_\mu \) is an instantaneous shape factor at size \( \mu \) and is not a mean over a range of sieve sizes. \( k_\mu \) is independent of the size distribution of the ground coal, whereas any mean value of \( k \) would depend on the size distribution between the sieve sizes averaged. \( k_\mu \) may be dependent on the type of coal ground.

B. Ratio of Sieve Size to Microscope Size.

Let the surface area \( S \) of a particle of sieve size \( \mu \) be given by

\[
S = k_1 \mu^2
\]

and its volume be given by

\[
V = k_2 \mu^3
\]

Then

\[
dS = k_1 \mu^2 dN
\]

and

\[
dV = k_2 \mu^3 dN
\]

where \( dN \) represents the number of particles of size \( \mu + d\mu \). If it is assumed that \( k_1 \) and \( k_2 \) are constant over the short size range investigated

\[
\frac{S}{V} = \frac{k_1}{k_2} \frac{\int \mu^2 dN}{\int \mu^3 dN}
\]

Now \( S/V = S_o \), and from equation (2)

\[
k = S_o \mu = \frac{k_1}{k_2}
\]

Therefore,

\[
S_o = k \frac{\int \mu^2 dN}{\int \mu^3 dN}
\]
From equation (6),
\[ \frac{d_p}{h} = f \]

Therefore,
\[ f = \frac{S_o}{k} \frac{\int_0^N d_p^3 dN}{\int_0^N d_p^2 dN} \]  \hspace{1cm} (16)

For a given sieve size fraction measured and counted, \( S_o \) is known from permeability measurements and \( k \) is known from equation (15). The integrals can be evaluated graphically by plotting \( d_p^2 \) and \( d_p^3 \) against cumulative number and taking the appropriate areas under the curves.

Values of \( f \) were also calculated using a volume mean microscope diameter and a volume mean sieve size for the sieve fractions of coal tested. These values agreed with those calculated using equation (16) for the cases where the nominal sieve size was accurate. However, as discussed in VI B, some of the sieves were very inaccurate and this led to erroneous values of \( f \). If instead of using the nominal sieve sizes, sizes were used which made the sieving results fit the best lines of Figure 3, the values of \( f \) were again in agreement with those calculated using equation (16). The use of equation (16) has two great advantages over the other methods: (a) The value of \( f \) does not depend on the sieve accuracy. (b) No concept of "mean diameter" is used, and the difficulty of deciding whether an area mean or a volume mean should be used is avoided.

C. Calculation of Weight Versus Sieve Size Distributions for Sub-Sieve Coal Fraction from Microscopic Measurements.

From microscopic count measurements, the cumulative per cent number of
particles, \( N \), below a given microscope size was determined as a smooth function of microscope size, \( d_p \). By plotting \( N \) against \( d_p^3 \), the percentage weight \( p \) below a given size \( d_p \) was obtained graphically since

\[
(p \rangle_{d_p} = 100 \int_0^{d_p^3} \frac{dp}{dN} \int_0^{d_p^3} \frac{dN}{dp} dp
\]

The microscope size was then converted to sieve size using the value of \( f \) previously determined, assuming that \( f \) had the same value for the sub-sieve fraction as found for the sieve fractions.

D. Compilation of Accurate Sieve Size Versus Percentage Weight Undersize Curves.

When two samples of the same coal are ground and sieved in a standard manner, the resultant size weight distributions may be different for two reasons: (a) The variability of the coal sample and grinding and (b) the variability of the sieving. However, if the percentage below a certain size is plotted against the number of revolutions of grinding, a smooth curve should be obtained. By drawing the best-fit family of smooth curves over a wide range of revolutions and sizes, the value picked off the curve for a definite number of revolutions will be more accurate. If the results are then plotted as a family of curves of percentage weight undersize against size, again the curves should be smooth; and results taken from the best-fit curve for a given number of revolutions should be more accurate. By successive cross-plotting and smoothing out discrepancies in this way, accurate size distributions are obtained.

This procedure was used to compile Figure 5 from the original
E. **Surface Area Change on Grinding.**

The surface area of ground coal was determined from the percentage weight versus size distribution, the values of shape factor k, and the density of the coal. If \( p \) is the percentage by weight below size \( \mu \), then the experimental data on size distribution may be expressed graphically in the form

\[
p = F(\mu), \quad \text{(some function of } \mu)\]

From equation (15)

\[
dS = dp \frac{k}{\mu p}
\]

Therefore, the hydrodynamic area of coal between \( \mu_1 \) and \( \mu_2 \) is given by

\[
S_{\mu_1-\mu_2} = \int_{\mu_1}^{\mu_2} k \, \frac{dp}{\mu p} \quad \text{(18)}
\]

or

\[
S_{\mu_1-\mu_2} = 2.3 \int_{\mu_1}^{\mu_2} \frac{k p}{\rho \mu} \, d(\log p) \quad \text{(19)}
\]

From the experimental plot of \( \log p \) against \( \log \mu \), \( p \), \( \mu \), and \( \log p \) may be obtained for any value of \( \mu \); and since \( k \) and \( p \) are also known for this size, \( \frac{kp}{\rho} \) may be plotted against \( \log p \) and \( S \) determined from the area under the curve.

Alternatively, if

\[
\frac{d(\log p)}{d(\log \mu)} = n
\]

(20)
This is somewhat more convenient as it allows a direct integration between any required sieve sizes. The values of \( n \) for any value of \( \log \mu \) are determined by taking the slope of the \( \log \rho/\log \mu \) distribution plot at that point. Below sieve sizes of 200 microns, \( n \) was found to be constant.

Over the part of the distribution for which \( n \) is a constant, \( p \) may be determined from the slopes and intercepts of the curve. Then

\[
S_{n-\mu} = 2 \int \frac{d^2}{d\mu^2} \left( \int \frac{d\rho}{\rho} \left( \frac{d\mu}{\mu} \right)^n \right) d\rho
\]

(21)

\[
P = \frac{d\rho}{\rho} \left( \int \frac{d\mu}{\mu} \right)^n
\]

(22)

where \( n \) and \( B \) may be determined from the slopes and intercepts of the distribution curve. Then

\[
S_{n-\mu} = \int k \rho B n \mu d\rho
\]

(23)

When \( k \) and \( \rho \) are constant,
F. **Breakage and Selection Mechanisms.**

(i) **Breakage mechanism.** Consider a particle of sieve size \( y \). On breakage in the mill, the particle will break into a size distribution such that a fraction by weight of \( q \) is less than a given sieve size \( x \), where \( x \leq y, q < 1 \). The relation between \( q \) and \( x \) defines the breakage mechanism of the particles of size \( y \) and the relations between \( q, x, \) and \( y \) is the breakage mechanism of the coal. The breakage mechanism is conveniently described by

\[
\left( \frac{dq}{d\mu} \right)_x = \frac{x^k y}{\mu} = f(x, y)
\]

where \( x^y \) is the weight fraction of the original particle in sieve size \( x + d\mu \), the original particle having a sieve size of \( y \).

(ii) **Selection mechanism.** The selection mechanism is concerned with the probability of breakage of particles of size \( y \) per revolution of grinding, \( \Pi_y \) say. The breakage is given by:

\[
\text{Breakage of particles of size } y + d\mu \text{ in } dR \text{ revolutions} = \Pi_y (dp)_y dR
\]

where \( (dp)_y \) is the fractional amount by weight of material in size \( y + dy \).

The selection mechanism is then defined by

\[
\Pi_y = F(y)
\]

(iii) **Combination of mechanisms.** Consider material of size \( x + d\mu \).

On grinding by a differential amount of grinding \( dR \), the amount of material...
of this size will be increased by contributions from breakage of material larger than $x$ but will be decreased by grinding out of material of size $x + dx$. The contribution of material of size $y$ per revolution is $\frac{dT_y}{dT_x} \left( \frac{dp}{d\mu} \right)_x$, which from equations 24, 25, is given by 

$$
\int \left[ d(\Delta_p) \right]_y = \Pi_y \frac{dp_y}{d\mu} k_y d\mu
$$

The contribution of all sizes above $x$ is therefore 

$$
\frac{dp_x}{d\mu} = \int_{y=x}^{y=y_u} \Pi_y x k_y dp_y
$$

where $y_u$ is the upper size limit of the material being ground. The grinding out of size $x$ per revolution is 

$$
\frac{dp_x}{d\mu} = -\Pi_x \left( \frac{dp}{d\mu} \right)_x
$$

Therefore, the total change is 

$$
\frac{dp_x}{d\mu} = \int_{y=x}^{y=y_u} \Pi_y x k_y dp_y - \Pi_x \left( \frac{dp}{d\mu} \right)_x
$$

Since $\left( \frac{dp}{d\mu} \right)_x$ and $dp_y$ change with revolutions of grinding and the above values are all for unit revolutions of grinding, they must be expressed in a differential form with respect to revolutions of grinding, i.e.

$$
\frac{\partial (\frac{dp}{d\mu})}{\partial R} = \int_{y=x}^{y=y_u} \Pi_y x k_y dp_y \partial R - \Pi_x \left( \frac{dp}{d\mu} \right)_x \partial R
$$

or 

$$
\frac{\partial^2 p_x}{\partial \mu \partial R} = \int_{y=x}^{y=y_u} \Pi_y x k_y dp_y - \Pi_x \left( \frac{dp}{d\mu} \right)_x
$$

(27)

If $\Pi_y$, $\Pi_x$, $x_k$ are constants with grinding, i.e. particles of coal of size $y$ break down in the same way with the same probability at 100 revolutions
as at 10 revolutions, then (27) can be represented as

$$\frac{\partial^2 \rho_k}{\partial \mu \partial \rho} = \sum_{y=x}^{y=y_k} K_y \Delta p_y - \Pi_x \left( \frac{\partial p}{\partial \mu} \right)$$ \hspace{1cm} (28)

where $K_y = \frac{\partial^2 \rho}{\partial \mu \partial \rho} y$ and since $\frac{\partial^2 \rho}{\partial \mu \partial \rho}$ and $\left( \frac{\partial p}{\partial \mu} \right)_{x}$ are known for various revolutions of grinding, mean values of $K_y$ can be estimated by considering the material greater than $x$ as several groups of $\Delta p$ within groups of size ranges. For example, if $x = 400$ microns, the groups chosen might be $400-700$ microns, $700-900$ microns, and $900-1190$ microns. Values of $\Delta p$ for these groups for various revolutions of grinding are known, and values of $400^{K}400\text{-}700^{K}$, $400^{K}700\text{-}900^{K}$, $400^{K}900\text{-}1190^{K}$, and $\Pi_{400}$ can theoretically be determined by solving the four simultaneous equations formed by taking values for $\frac{\partial^2 \rho_{400}}{\partial \mu \partial \rho}$, $\left( \frac{\partial p}{\partial \mu} \right)_{400}$, $\Delta p_{400\text{-}700}$, $\Delta p_{700\text{-}900}$, $\Delta p_{900\text{-}1190}$ for four different revolutions in grinding. In general, these values will not be known with sufficient accuracy to calculate values of $K$ algebraically; but a crude estimate of the values may be obtained by the following method.

Let

$$\frac{\partial^2 \rho}{\partial \mu \partial \rho} = \frac{K_{400}^{K}400\text{-}1190^{K}}{400^{K}400\text{-}1190} \Delta p_{400\text{-}1190} - \Pi_{400} \left( \frac{\partial p}{\partial \mu} \right)_{400}$$ \hspace{1cm} (29)

$400^{K}700\text{-}1190^{K}$ will not be a constant with revolutions of grinding but by assuming that it is and plotting

$$\frac{\partial^2 \rho}{\partial \mu \partial \rho} \frac{\partial \rho}{\partial \mu} \text{ against } \frac{\partial p_{400\text{-}1190}}{\partial \mu} \frac{\partial \rho}{\partial \mu},$$

the best line through the
points gives an estimate of $\Pi_{400}$ as the negative intercept on the \( \frac{\partial^2 P}{\partial \mu \partial R} / \frac{\partial P}{\partial \mu} \) axis.

Then let

\[
\frac{\partial^2 P}{\partial \mu \partial R} + \Pi_{400} \frac{\partial P}{\partial \mu} = \frac{400}{400} K_{400-900} \frac{\Delta P_{400-900}}{\Delta P_{900-1190}} + \frac{400}{400} K_{900-1190}
\] (29a)

will not be a constant but assuming it is and plotting the L.H.S. of equation 29a against $\Delta P_{400-900}$, the best fit through the points gives an estimate of $400 K_{900-1190}$ as an intercept. This procedure repeated will give estimates for all the K's. Clearly such a procedure only works when the plots approach straight lines.

(iv) Relation with the breakage mechanism proposed by Calcott (20).

Calcott has proposed a breakage mechanism defined by

\[
q_x = \frac{1 - e^{-\frac{x}{y^2}}}{1 - e^{-1}}
\] (30)

At a fixed value of $x$, where the variable $\mu$ is the variable $y$,

\[
\left( \frac{\partial q_x}{\partial \mu} \right)_x = 1.58 \frac{x^2}{y^2} e^{-\frac{x}{y^2}}
\] (31)

By comparison with equation (24), it will be seen that

\[
x y k' = 1.58 \frac{x^2}{y^2} e^{-\frac{x}{y^2}}
\]

The values of $x y$ estimated are $k' \Pi_y$. Therefore,
\[
\frac{x K_y}{y} = 1.58 \frac{x}{y^2} e^{x y} 
\]  
(32)

Therefore, knowing \( y \), the values of \( K \) estimated from the experimental data may be compared with those proposed by Calcott.

VI. EXPERIMENTAL RESULTS

A. Characteristics of Coal.

The proximate and ultimate analyses of the coals tested are given in Table I. The four coals cover a range from low rank high volatile sub-bituminous coal to high rank low volatile anthracite.

B. Size/Weight Distributions of Ground Coals.

Figure 3 gives the original distribution curves of coal B-19447 ground for varying revolutions. Table 2 gives the distribution results for four coals ground by the Hardgrove standard test method (60 revolutions).

It can be seen from Figure 3 that points for some of the sieves fall consistently above or below the line of best fit. Clearly the true sieve size is different in these cases from the nominal sieve size. Duplicate runs will not in general give exactly the same distribution. To obtain more consistent results, the procedure described in V C was followed. Taking points from the best-fit lines in Figure 3 and cross plotting them as functions of varying revolutions of grinding gave Figure 4. Points from the best-fit lines in Figure 4 were used to compile Figure 5, which was used
EXPERIMENTAL PERCENTAGE WEIGHT UNDERSIZE VERSUS SIEVE SIZE DISTRIBUTIONS FOR COAL B-19447 GROUND FOR VARYING REVOLUTIONS

Figure 3
VARIATION OF % WEIGHT UNDERSIZE WITH REVOLUTIONS OF GRINDING FOR COAL B-19447

Figure 4
CORRECTED PERCENTAGE WEIGHT UNDERSIZE VERSUS SIEVE SIZE DISTRIBUTIONS FOR COAL B-19447

Figure 5
\[
\begin{align*}
    H &= \frac{H_2 + 0.015S - 0.01A(p)}{(C - 0.17)p} = \text{Moisture} \\
    C &= \frac{(C - 0.17)p}{p} = \text{Carbon} \\
    \text{Volatile Matter, D.A.} &= \frac{100 - H_2 - 0.08A - 0.25S}{100} = \text{Nitrogen}
\end{align*}
\]

## TABLE 1

<table>
<thead>
<tr>
<th>Coal</th>
<th>Constituent As Part's</th>
<th>% Basis Used</th>
<th>% Basis % Basis Used</th>
<th>% Basis Used</th>
<th>% Basis % Basis Used</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B-17790</td>
<td>B-19426</td>
<td>Ste. Nicholas Anthracite</td>
<td>B-17790</td>
<td>B-19426</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>2.6</td>
<td>4.7</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.88</td>
<td>1.47</td>
<td>1.52</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.37</td>
<td>3.9</td>
<td>5.10</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>95.3</td>
<td>80.6</td>
<td>78.8</td>
<td>87.6</td>
<td></td>
</tr>
<tr>
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<td>0.51</td>
<td>1.175</td>
<td>1.75</td>
<td>2.9</td>
<td>4.5</td>
</tr>
<tr>
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<td>4.5</td>
<td>17.9</td>
<td>11.2</td>
<td>7.6</td>
<td></td>
</tr>
<tr>
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<td>9.1</td>
<td>14.4</td>
<td>7.9</td>
<td>16.4</td>
<td></td>
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<td></td>
<td>1.6</td>
<td>0.5</td>
<td>0.8</td>
<td>1.5</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 2

SIZE-WEIGHT DISTRIBUTIONS AND HYDRODYNAMIC SURFACE AREAS OF SIEVE FRACTIONS FOR COALS GROUND ACCORDING TO THE STANDARD HARDGROVE TEST

<table>
<thead>
<tr>
<th>Sieve Range</th>
<th>B-19447</th>
<th>B-17990</th>
<th>B-19426</th>
<th>S.N.A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Standard Mesh</td>
<td>p</td>
<td>S_o</td>
<td>p</td>
<td>S_o</td>
</tr>
<tr>
<td>16 x 30</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>30 x 35</td>
<td>64.25</td>
<td>178</td>
<td>79.97</td>
<td>138</td>
</tr>
<tr>
<td>35 x 50</td>
<td>54.05</td>
<td>235</td>
<td>71.78</td>
<td>217</td>
</tr>
<tr>
<td>50 x 70</td>
<td>35.55</td>
<td>403</td>
<td>54.06</td>
<td>307</td>
</tr>
<tr>
<td>70 x 100</td>
<td>27.02</td>
<td>565</td>
<td>44.78</td>
<td>439</td>
</tr>
<tr>
<td>100 x 120</td>
<td>18.48</td>
<td>710</td>
<td>35.62</td>
<td>598</td>
</tr>
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<td>120 x 140</td>
<td>16.89</td>
<td>865</td>
<td>32.78</td>
<td>714</td>
</tr>
<tr>
<td>140 x 170</td>
<td>14.63</td>
<td>1001</td>
<td>28.78</td>
<td>948</td>
</tr>
<tr>
<td>170 x 200</td>
<td>12.95</td>
<td>1171</td>
<td>25.95</td>
<td>1036</td>
</tr>
<tr>
<td>200 x 230</td>
<td>11.17</td>
<td>1403</td>
<td>22.26</td>
<td>1235</td>
</tr>
<tr>
<td>230 x 325</td>
<td>9.82</td>
<td>1730</td>
<td>20.26</td>
<td>1635</td>
</tr>
<tr>
<td>Minus 325</td>
<td>7.94</td>
<td>11.14</td>
<td>16.29</td>
<td>3.20</td>
</tr>
<tr>
<td>% Weight loss on grinding</td>
<td>0.66</td>
<td>0.72</td>
<td>0.83</td>
<td>(-0.1)</td>
</tr>
<tr>
<td>Mean Hardgrove Index</td>
<td>52</td>
<td>93</td>
<td>99</td>
<td>30</td>
</tr>
</tbody>
</table>

p = % by weight below upper sieve size

S_o = Surface area per cubic cm. of coal in size range given: cm.²/cm.³
for the integrated surface area determinations described in VI E. It can be seen that the points in Figure 5 are very consistent and the distribution curves can be drawn with considerable accuracy. If the straight line portions of the curves are extrapolated and the values of \( n \) and \( \beta \) (see equation 22) determined, then the results are again very consistent, as can be seen from Figure 6. It will be noted, however, that the extrapolation of the \( B \) curve to zero revolutions does not give a value of \( B \) of zero which is contrary to theoretical expectations (see VII B). Similar curves were obtained for coal B-17990.

C. Moisture Change During Grinding.

Table 3 gives the moisture contents of coal B-19447 originally and after grinding. Although the coal was completely dried before grinding, by the time the grinding and sieving operation was over, the ground coal had regained the moisture content of the original material. It must be concluded that for a coal in equilibrium with the atmosphere, the ground coal will hold as much moisture as the unground coal; and, hence, any weight loss on grinding is loss of fine coal and/or occluded gases.

D. Apparent Density of Coal Fractions.

The results of geometric or apparent density measurements on various coal fractions are shown in Figure 7. In general, the value of density determined at a given size varied about the mean line within a range of \( \pm 3\% \). Fortunately, however, the value of \( \frac{ds}{dp} \) in equation (13) is insensitive to small changes in density; and the effect of density on shape factor is mainly the direct proportionality shown in equation (15). It is interesting to see that the water densities were the same as the mercury densities.
VALUES OF "n" AND "B" IN $p = B\mu^n$

FOR COAL B-19447 GROUND FOR VARYING REVOLUTIONS

Figure 6
<table>
<thead>
<tr>
<th>Sample</th>
<th>Moisture content gms. per 100 gms. of original material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>1.25</td>
</tr>
<tr>
<td>After drying and immediately before grinding</td>
<td>0</td>
</tr>
<tr>
<td>After grinding and sieving</td>
<td>1.27</td>
</tr>
</tbody>
</table>
APPARENT DENSITIES OF SIEVE FRACTIONS OF COALS
GROUND ACCORDING TO STANDARD HARDGROVE TEST

Figure 7
within the limits of accuracy of both methods.

It can be seen that for the softer coals the larger size fractions remaining on grinding have a higher density, and the smaller fractions have a lower density. This is almost certainly due to concentration of dense mineral matter in the coarse fractions (15) and depletion in the fine fractions, since the mineral matter is less easily ground. This effect is more marked in the softer coals for two reasons. (a) The mineral matter is comparatively harder for these coals. (b) The grinding has proceeded to a more advance stage for the softer coals (although ground for the same number of revolutions), and clearly the concentration of mineral matter will be a function of the degree of grinding. As will be seen later (Section VI I), this process is somewhat counter-balanced by the fact that larger particles have a greater probability of being ground.

E. Area-to-Volume Shape Factors.

Table 4 gives the values of specific surface area determined by the permeability apparatus for the sieve fractions of the four coals tested. These values are the mean of at least three results. Triplicate runs gave values which were always within ±3% of the mean value. Figure 8 shows the cumulative surface area against cumulative percentage weight curves for the four coals. Table 4 gives the values of dS/dp at selected sieve sizes calculated from the slopes of the curves using equation (13), and the values of the shape factors calculated using equation (15). The values of k over the sieve size range investigated, approximately 40 to 600 microns, were found to be constant for a given coal, the values varying randomly about
TABLE 4
SURFACE AREA TO VOLUME SHAPE FACTORS FOR COALS GROUND ACCORDING TO STANDARD HARDGROVE TEST

<table>
<thead>
<tr>
<th></th>
<th>B-19447</th>
<th></th>
<th>B-17990</th>
<th></th>
<th>B-19426</th>
<th></th>
<th>St. Nicholas Anthracite</th>
</tr>
</thead>
<tbody>
<tr>
<td>µ</td>
<td>ds/dp</td>
<td>k</td>
<td>µ</td>
<td>ds/dp</td>
<td>k</td>
<td>µ</td>
<td>ds/dp</td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td>-------</td>
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<td>--------</td>
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<td>-------</td>
</tr>
<tr>
<td>47.5</td>
<td>1421</td>
<td>9.6</td>
<td>44</td>
<td>1420</td>
<td>7.9</td>
<td>44</td>
<td>1240</td>
</tr>
<tr>
<td>62</td>
<td>1124</td>
<td>9.6</td>
<td>62</td>
<td>1040</td>
<td>8.1</td>
<td>62</td>
<td>900</td>
</tr>
<tr>
<td>74</td>
<td>935</td>
<td>9.5</td>
<td>74</td>
<td>850</td>
<td>8.0</td>
<td>74</td>
<td>809</td>
</tr>
<tr>
<td>88</td>
<td>782</td>
<td>9.5</td>
<td>88</td>
<td>726</td>
<td>8.0</td>
<td>88</td>
<td>632</td>
</tr>
<tr>
<td>105</td>
<td>678</td>
<td>9.8</td>
<td>105</td>
<td>610</td>
<td>8.2</td>
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<td>125</td>
<td>565</td>
<td>9.7</td>
<td>125</td>
<td>480</td>
<td>7.8</td>
<td>125</td>
<td>486</td>
</tr>
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<td>138</td>
<td>497</td>
<td>9.6</td>
<td>149</td>
<td>396</td>
<td>7.9</td>
<td>149</td>
<td>357</td>
</tr>
<tr>
<td>220</td>
<td>338</td>
<td>9.8</td>
<td>220</td>
<td>268</td>
<td>8.0</td>
<td>210</td>
<td>250</td>
</tr>
<tr>
<td>297</td>
<td>230</td>
<td>9.4</td>
<td>297</td>
<td>197</td>
<td>8.0</td>
<td>297</td>
<td>173</td>
</tr>
<tr>
<td>500</td>
<td>140</td>
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<td>500</td>
<td>128</td>
<td>8.3</td>
<td>500</td>
<td>100</td>
</tr>
<tr>
<td>590</td>
<td>118</td>
<td>9.6</td>
<td>590</td>
<td>104</td>
<td>7.9</td>
<td>690</td>
<td>87.7</td>
</tr>
</tbody>
</table>

Mean k = 9.6

Key
µ is sieve size in microns
\( \frac{ds}{dp} \) is instantaneous specific surface area in cm.\(^2\) per gram.

k is volume-to-surface area shape factor
the mean within about ±4%. There were marked differences in the shape factors found for the four different coals, these differences being greater than could be explained by the experimental error in the various determinations made. In the following work, it is assumed that the shape factor for a given coal is constant over the complete size range investigated.

F. Microscopic Size to Sieve Size Factor.

Table 5 gives the ratio of the microscope diameter to the sieve size, determined using equation (16). Figure 9 is an example of the curves used to determine the value of \( f \). There is no apparent trend of change of ratio with size, and also the ratio is not significantly different between the coals. It was noted that there was a tendency for a particular operator to obtain consistently higher or lower results than the mean; therefore, counts were performed by at least two different people. The mean value of \( f \) for all the results was 1.68; and any given value fell within about ±4% of this mean irrespective of the sieve size range, type of coal, or person conducting the count.

G. Size Distribution Below 325 Mesh.

Table 6 gives the cumulative weight versus microscope size (and corresponding sieve size) calculated using equation (17), expressed both as a percentage of the minus 325 mesh sample tested and as actual weight of the minus 325 sample. The weight loss on grinding (60 revolutions) and sieving was 0.66 gms. per 100 gms., and it was assumed that this loss was in very fine material. Therefore, it was added to the cumulative weight down to 1 1/2 microns. Figure 10 shows a complete sieve size/weight distribution for the coal tested, using a factor of 1.68 to divide
TABLE 5
RATIO OF MICROSCOPE PROJECTED-AREA-DIAMETER TO SIEVE SIZE

<table>
<thead>
<tr>
<th>Coal</th>
<th>Sieve Range U.S. Mesh</th>
<th>Sieve Size Microns</th>
<th>$f = \frac{r_d}{\mu}$</th>
<th>Mean value of $f$</th>
<th>For all coals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>For sieve fraction</td>
<td>For coal</td>
</tr>
<tr>
<td>B-19447 (60 revs.)</td>
<td>325 x 230</td>
<td>44-62</td>
<td>1.75</td>
<td>1.66</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>230 x 200</td>
<td>62-74</td>
<td>1.93</td>
<td>1.69</td>
<td>1.68</td>
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<td></td>
<td>200 x 170</td>
<td>74-88</td>
<td>1.96</td>
<td>1.69</td>
<td>1.68</td>
</tr>
<tr>
<td>S.N.A. (60 revs.)</td>
<td>325 x 230</td>
<td>44-62</td>
<td>1.72</td>
<td>1.64</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>230 x 200</td>
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<td>1.70</td>
<td>1.72</td>
<td>1.70</td>
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<tr>
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<td>200 x 170</td>
<td>74-88</td>
<td>1.64</td>
<td>1.69</td>
<td>1.69</td>
</tr>
<tr>
<td>B-17790 (60 revs.)</td>
<td>325 x 230</td>
<td>44-62</td>
<td>1.61</td>
<td>1.61</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>230 x 200</td>
<td>62-74</td>
<td>1.95</td>
<td>1.71</td>
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</table>
CUMULATIVE SURFACE AREA AGAINST PERCENTAGE UNDERSIZE FOR FOUR COALS
GROUND ACCORDING TO STANDARD HARGROVE TEST

Figure 8
EXAMPLE OF CURVES USED TO CALCULATE
VALUES OF $f$

Figure 9
TABLE 6

SIZE DISTRIBUTION BELOW 325 MESH OF COAL B-19447

GROUND ACCORDING TO STANDARD HARDGROVE TEST

<table>
<thead>
<tr>
<th>Microscope diameter, microns</th>
<th>Equivalent sieve size, microns</th>
<th>Cumulative % by weight of -325 fraction tested</th>
<th>Cumulative weight expressed as a % of total coal ground</th>
<th>Plus 0.66% weight loss of fine material on grinding</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.9</td>
<td>0.52</td>
<td>0.0143</td>
<td>0.00104</td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td>0.78</td>
<td>0.049</td>
<td>0.00356</td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td>1.04</td>
<td>0.148</td>
<td>0.0108</td>
<td></td>
</tr>
<tr>
<td>2.6</td>
<td>1.56</td>
<td>0.56</td>
<td>0.0408</td>
<td>(0.700)</td>
</tr>
<tr>
<td>3.5</td>
<td>2.08</td>
<td>1.24</td>
<td>0.090</td>
<td>(0.750)</td>
</tr>
<tr>
<td>4.4</td>
<td>2.6</td>
<td>2.18</td>
<td>0.159</td>
<td>(0.819)</td>
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<td>5.8</td>
<td>3.47</td>
<td>3.67</td>
<td>0.267</td>
<td>0.927</td>
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<td>7.3</td>
<td>4.34</td>
<td>6.00</td>
<td>0.436</td>
<td>1.096</td>
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<td>8.75</td>
<td>5.2</td>
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<td>10.2</td>
<td>6.06</td>
<td>9.90</td>
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<td>14.7</td>
<td>1.07</td>
<td>1.73</td>
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<td>12.5</td>
<td>29.2</td>
<td>2.12</td>
<td>2.78</td>
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<td>26</td>
<td>15.6</td>
<td>36.1</td>
<td>2.62</td>
<td>3.28</td>
</tr>
<tr>
<td>25</td>
<td>20.8</td>
<td>44.9</td>
<td>3.26</td>
<td>3.92</td>
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<tr>
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<td>56.0</td>
<td>4.07</td>
<td>4.73</td>
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<tr>
<td>52</td>
<td>31.0</td>
<td>70.6</td>
<td>5.14</td>
<td>5.80</td>
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<tr>
<td>61</td>
<td>36.4</td>
<td>83.8</td>
<td>6.10</td>
<td>6.76</td>
</tr>
<tr>
<td>70</td>
<td>41.6</td>
<td>93.7</td>
<td>6.82</td>
<td>7.48</td>
</tr>
<tr>
<td>79</td>
<td>47</td>
<td>100.0</td>
<td>7.28</td>
<td>7.94</td>
</tr>
</tbody>
</table>
microscope size to give sieve size (see VI F above). It can be seen that over the range 3 to 300 microns, the distribution is a straight line on the log/log plot. This type of distribution has been noted previously (14, 15, 16) and extended below sieve sizes by air elutriation. Rosin and Rammler (23) used air elutriation to extend results to sub-sieve sizes and concluded that the distribution obeyed the Rosin-Rammler law. However, for small sieve sizes, the Rosin-Rammler distribution becomes the simple power distribution found in Figure 10. The departure of the curve from the straight line below three microns is almost certainly due to the assumption that all of the weight loss on grinding is less than 1 1/2 microns. The break in the curve is clear evidence that the weight loss is in sizes mainly below three microns (sieve size).

In the following sections, it is assumed that the distribution of fine coal is a straight line on log/log paper, down to the finest sizes produced on grinding.

H. Change of Hydrodynamic Surface Area with Number of Revolutions of Grinding.

Figure 11 shows the surface areas of the ground coal fractions for coal B-19447 calculated using equations (19) and (23). The surface areas were calculated assuming that the log p/log $\mu$ distribution found in G above could be extrapolated to a lower limit of 1, 0.1, or 0.01 microns and also assuming that the shape factor determined from 40 to 600 microns was also constant down to the lower size limit. It is clear that the lower size limit chosen considerably affects the absolute value of the hydrodynamic surface area. The curve is a straight line only when a lower size limit of between 0.8 and 1 micron is used. For lower size limits of 0.1 and 0.01 microns, the area increases more with revolutions in grinding than
EXTENDED PERCENT WEIGHT UNDERSIZE VERSUS SIEVE SIZE DISTRIBUTION FOR COAL B-19447 GROUND FOR 60 REVOLUTIONS

Figure 10
INCREASE OF HYDRODYNAMIC SURFACE AREA OF GROUND COAL WITH REVOLUTIONS OF GRINDING FOR COAL B-19447, ASSUMING 0.1 AND 0.01 MICRONS AS THE SMALLEST SIZE PRESENT.

Figure 11
Rittinger's Law would predict. Another feature of interest is that the extrapolation of the curves to zero revolutions gives an initial surface area of unground material of 1.2 sq. meters, whereas the actual unground surface area is 0.8 sq. meter. It appears that an initial small amount of grinding produces 0.4 sq. meter of surface area in addition to the 0.117 sq. meter per 100 gms. per revolution produced for the remainder of the grinding process.

Figure 12 shows the surface area change with grinding for coal B-17790, where a lower limit of 1 micron has been used. After an increase to about 16 sq. meters per 100 gms., it appears that the increase in surface area is no longer proportional to the revolutions of grinding. This conclusion is based on one test result only and must be verified in future work. Extrapolation of the straight line portion of the curve to zero revolutions again indicates an initial area of 1.2 sq. meters instead of the 0.8 sq. meter of the unground sample.

Table 7 gives the surface areas from 1 micron to 1190 microns for the four coals ground in the Hardgrove test and also gives the increase in surface areas on grinding. For coal B-19426 and the St. Nicholas Anthracite the results are based on measurements at 60 revolutions only; the cross plotting technique was not used as the data were not available.


Figure 13 shows the values of \( \frac{\partial p}{\partial \mu} \) as a function of sieve size and revolutions of grinding for coal B-19447, the values being obtained by graphical differentiation of Figure 5. Figure 14 shows the values of \( \frac{\partial p}{\partial \mu \partial R} \) for the same coal and variables, derived by graphical differentiation of
Assuming 1 micron as lower limit.

Figure 12

Increase of hydrodynamic surface area of ground coal with revolutions of grinding for coal B-17790.

Revolutions of Grinding

Surface Area of 100 gms Ground Coal

meters$^2$/100 gms

0 2 4 6 8 10 12 14 16 18 20 22 24
TABLE 7

HYDRODYNAMIC SURFACE AREAS OF COALS
GROUND ACCORDING TO THE STANDARD HARDGROVE TEST

<table>
<thead>
<tr>
<th>Coal</th>
<th>Hardgrove Grindability Index</th>
<th>Surface Area Ground Coal m.²/100 gms.</th>
<th>Increase in Surface Area m.²/100 gms.</th>
<th>Rank-Index of Coal % C-8.5 %H</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-19447</td>
<td>52</td>
<td>7.6</td>
<td>6.8</td>
<td>33.5</td>
</tr>
<tr>
<td>B-17790</td>
<td>93</td>
<td>15.9</td>
<td>15.1</td>
<td>44.2</td>
</tr>
<tr>
<td>B-19426</td>
<td>99</td>
<td>14.5</td>
<td>13.7</td>
<td>52.4</td>
</tr>
<tr>
<td>St. Nicholas Anthracite</td>
<td>30</td>
<td>3.4</td>
<td>2.6</td>
<td>77.0</td>
</tr>
</tbody>
</table>
VALUES OF $\frac{\partial \theta}{\partial \mu}$ FOR VARYING REVOLUTIONS OF GRINDING,
FOR A FAMILY OF SIEVE SIZES (COAL B-19447)

Figure 13
VALUES OF $\frac{\delta^2 p}{\delta \mu \delta R}$ FOR VARYING REVOLUTIONS OF GRINDING (COAL B-19447)

Figure 14
VII. DISCUSSION OF RESULTS

A. Rittinger's Law.

In spite of the theoretical objections to Rittinger's Law there is considerable evidence that under a restricted range of conditions the Law is true to a close degree of accuracy. The correlation of the increase in surface area with grinding obtained in this work is not conclusive, since the lower size limit chosen for the integration to obtain surface area is quite arbitrary. The microscopic studies of the fine fractions of ground coal indicated that material below 1 micron in size was not present in large quantities\(^1\), although there still remains the question of the fineness of the material making up the weight loss on grinding. Assuming a lower limit of 0.1 micron, Figure 11 indicates that the surface area per revolution increases at high degrees of grinding. This is unreasonable on theoretical grounds, since one would expect that as the material being ground became finer it would become increasingly difficult to grind and would thus require more energy for the production of broken surface rather than less. By taking a lower limit of 1 micron, it is not assumed that material less than 1 micron is absent but rather that 1 micron represents an effective lower limit for the straight line log p/log µ distribution extrapolated from the sieving results. The strict linearity, over a wide range of grinding, of the results plotted in Figure 11 for the 1 micron lower limit would hardly occur by

\(^1\)From electron micrographs of ground coal, Preston and Cuckow (32) concluded that coals ground in the normal manner had few particles of less than about 1 micron in size.
coincidence; and it must be concluded that the evidence for the accuracy of Rittinger's Law is quite strong.

One of the principal theoretical objections to the Law may be overcome by considering the fracturing of the coal to occur in the following manner. If a particle of coal is crushed it must be raised to a strained state before it fractures. On fracture the energy of the strained state will be expended in two ways. (1) It will be used to break chemical bonds to give the fresh surface of the broken material and (2) it will be released as heat of fracture. This process may be considered as analogous to the fracture of a brittle spiral spring loaded beyond its breaking point. The spring will break at a flaw, and energy will be used to break the chemical bonds across the fracture. Also each half of the spring will contract to an unstrained state, releasing energy as heat. In addition to these two processes, in the grinding of coal there is the probability of the loss of energy as frictional heat due to slip over the ground material. Rittinger's Law states that the fraction of the energy used to break chemical bonds is a fixed fraction of the total energy expended. For a given charge of coal ground at a fixed speed under a fixed load, the fraction of energy expended in fractional slip is quite likely to be constant. A distinction must now be made between the "strength" of a coal and the "hardness" of a coal. The strength is here arbitrarily defined as the strain energy required before the particle fractures (which is a function of the type of forces imparting this energy to the particle); whereas the hardness is arbitrarily defined as the strength of the chemical bonds in the material. Clearly two particles may have the same chemical composition and hence the same hardness but may
have widely differing strengths, if one is highly flawed and the other not. Consider two such particles of similar "chemical hardness" but different strengths. The stronger one will require the addition of more energy to fracture it, but it is conceivable that on fracture it will break into many smaller pieces. On the other hand the weaker particle will break more readily with a lower energy content but will break into fewer pieces with correspondingly lower fresh surface. Similarly, a large impulsive force of low energy application might cause breakage with small area production; whereas a smaller force applied for a much longer time and greater deformation would produce a larger surface area on eventual shatter. It is possible that particles of entirely different strengths with different probabilities of breakage have breakage functions which automatically compensate so that the fraction of the strain energy which is used to produce fresh surface is constant. Such a concept would clearly explain the validity of Rittinger's Law even although the probability of breakage is known to vary with size and degree of grinding.

As particles become smaller and stronger on grinding, they may eventually reach a state where the crushing forces of the machine are insufficient to cause much breakage. At this stage, the fraction of energy lost as frictional slip in the charge might be expected to increase. This is observed in Figure 12 for the softer and more highly ground B-17790 coal. This effect was also noted by Hardgrove (10), who explained it as being due to cushioning of the grinding process by the fines, an explanation which is virtually the same, although expressed differently, as the hypothesis suggested here.
Grinding experience (33) indicates that it is extremely difficult to reduce anthracite below 0.1 to 1 micron in size in conventional grinding apparatus. It may be postulated that somewhere near this size range the major flaw structure of the coal has been completely broken out, and grinding is more difficult by an order of magnitude or more. Van Krevelen (34) states that Boddy found coal particles to be initially crushed to 1 micron in size. As the surface area of this material is of the same order as the macropore area of unground coal, Van Krevelen suggests that breakage to 1 micron is favored by the macropore system. For smaller particles, the coal tends to plastically deform rather than fracture; this implies much greater strength and a low grindability.

It is interesting to note that Frick (35) states that the coefficient of friction of anthracite is much lower than for other coals, which leads to large energy loss as frictional slip. It is always assumed that the operation of grindability test machines imparts a standard quantity of energy to the coal charge. If the coefficient of friction is not the same for all coals, then the fresh surface produced per revolution of grinding will depend not only on the hardness of the coal but also on the fraction of energy lost by slip, consequently, the grindability indices obtained will not be a satisfactory index for mills such as ball mills and impact grinders.

B. Breakage and Selection Mechanisms.

Consider the variation of $\frac{\partial}{\partial \mu \sigma R}$ with revolutions of grinding. In general it might be expected that the curve would be as shown in Figure 15. Noting that $\frac{\partial}{\partial \mu \sigma R}$ is the increase in weight of material sieve size
Figure 15

With Degree of Grinding

\[ \frac{\delta p}{\delta R} \]

Diagrammatic Representation of Variation of

% Weight Per Micron

Per Revolution

Applies for a fixed size

of a fixed size.
\( \mu \), per revolution of grinding, point A on the curve of Figure 15 represents the initial rate of increase of material of size \( \mu \) per revolution where the amount is being increased due to breakdown of larger material. It would appear (see below in this section) that on breakage of a given size the major weight of material produced is in sizes a little below that of the original material. That is, breakage of 900 micron material will give a sizeable fraction of material between 800 and 900 microns in size. Thus, if on grinding, the size distribution of material greater than \( \mu \) is altered to give a product which on re-grinding will give a larger donation to material of size \( \mu \), then \( \frac{d^2}{d\mu^2} \) will increase. However, this trend is opposed by the continual decrease of the total amount of material greater than \( \mu \) which is available to be broken to \( \mu \) and also the material of size \( \mu \) being itself ground down to smaller sizes. Clearly as the amount of size \( \mu \) increases, the breakage of size \( \mu \) increases. Thus a point is reached where the rate of increase of \( \frac{d^2}{d\mu^2} \) is zero. This is point B. From point B to point C, the amount of material of size \( \mu \) is still increasing with grinding; but it is increasing at a progressively slower rate, until at C the increase in material of size \( \mu \) due to breakdown of larger sizes is exactly matched by the decrease due to breakage of size \( \mu \). From C to D the amount of material of size \( \mu \) decreases, that is, it is being ground down faster than it is replenished. From point D onwards, the amount is still decreasing but because there is less and less of material of size \( \mu \) to grind, its rate of grinding becomes progressively smaller. Consequently, \( \frac{d^2}{d\mu^2} \) asymptotically approaches zero (point E).

Figure 15 is the general case; but depending on the size considered,
the actual case may start at varying places along this curve. For instance, for the case of material of maximum size, 1190 microns, there is no possibility of increase of $\frac{\partial \gamma}{\partial u}$, as there is no greater sized material to break down to 1190 microns. Therefore, the amount must progressively decrease and $\frac{\partial \gamma}{\partial \theta \mu \rho} \mu \rho$ must be negative. Furthermore, the rate of decrease of $\frac{\partial \gamma}{\partial \theta \mu \rho} \mu \rho$ must progressively decrease with grinding. Therefore, the curve for the maximum size material must start at D for zero revolutions and progress towards E. Again for smaller sizes the production of more suitably sized larger material is overweighted by the decrease in total quantity of this larger material; therefore, the curves start after point B.

On examination of Figure 14, it will be seen that the variation of $\frac{\partial \gamma}{\partial \theta \mu \rho} \mu \rho$ with revolutions of grinding and size of material considered follows the general pattern expected from the above discussion. However, more detailed examination indicates that the behavior of the coal on grinding cannot be explained on the basis on equation (27), assuming that $\Pi_{yx} K_y$ is constant with revolutions of grinding. The inconsistencies found cannot be explained on the basis of experimental error, because although the curves of Figure 14 cannot be drawn with great accuracy they should be sufficiently precise to enable the determination of approximate values of $K_{xy}$ by the techniques described in section V F (iii). It becomes obvious on setting the results in the form of equation (28) that values of $K_{xy}$ must be greater for the initial stages of grinding for the equations to be consistent. However, using equation (29) and plotting $\frac{\partial \gamma}{\partial \mu \rho} \mu \rho / \frac{\partial \gamma}{\partial \mu} \mu$ against $\Delta_{\rho x-100} / \frac{\partial \gamma}{\partial \mu} \mu$, values of $\Pi_{x}$ were estimated, although the drawing
of a mean straight line through the points was very arbitrary in some cases. Figure 16 shows $\Pi_x$ plotted against $x$ on log/log scales; and it can be seen that although scatter is large, the probability of material of sieve size $x$ being selected for grinding is definitely dependent on the particle size and is approximately proportional to the $0.9$ power of $x$ or,

$$\Pi_x \propto x^q$$

(33)

This result is not unexpected since it would appear logical that in the ring-ball mill (where material is crushed between two moving surfaces) larger sizes should be preferentially crushed. Also larger sizes have a bigger probability of containing major flaws and would again break more readily. This second point leads to the probable explanation of the inconsistencies of values of $K_y$, and hence $\Pi_y$, with increase of grinding. As the coal sample is heterogenous in physical properties, it seems highly probable that in the initial stages of grinding the particles of size $\mu$ which break are the weakest of that size but as grinding proceeds the particles of size $\mu$ left are progressively stronger (but not necessarily harder) than those crushed at first. These remnants would include dense, strong particles of mineral matter and also denser and more highly coalified coal. Clearly a concentration of density in the coarser fractions would be expected on continued grinding, and this is indeed found (see Section VI D). That this concentration is not more pronounced is due to the fact that this coarser material has a higher probability of breakage than finer sizes due to the mechanics of the crushing operation. Considering Figure 14, it is seen that the values of $\theta^2/\theta_{\text{max}}^2$ may be extrapolated
RELATION OF PROBABILITY OF BREAKAGE PER REVOLUTION WITH SIEVE SIZE OF MATERIAL BROKEN

Figure 16
to zero revolutions with good accuracy for 600 microns and less material.

At zero revolutions, the values of \( \frac{\partial p}{\partial \mu} \) for material below 590 microns are theoretically zero; and equation (28) reduces to

\[
\frac{\partial^2 p}{\partial \mu \partial R} = \left( \Gamma_{590-1190} \cdot x_{590-1190} \right) 100 \tag{34}
\]

where \( \Gamma_{590-1190} \) is the mean probability of breakage per revolution of the unground 16 x 30 mesh sieve and \( x_{590-1190} \) is the fraction of the unground material which falls within the size \( x + \Delta x \) on breakage. Examination of Figure 13, however, shows that the values of \( \frac{\partial p}{\partial \mu} \) extrapolated to zero revolutions are not, in general, zero. This surprising result has two possible explanations. Firstly, it is possible that the sieving operation produces small quantities of minus 30 mesh material which appear as constant initial values of \( \frac{\partial p}{\partial \mu} \). Secondly, it is possible that the initial sample for grinding contains particles which are just too strong to break down on sieving but are so weak that the first revolution of the mill fractures them, even if they are not directly in the grinding area.

Whatever the explanation of its presence, this finer material present at "zero" revolutions explains why the value of surface area obtained by extrapolating Figures 11 and 12 back to zero revolutions (see Section VI H) is higher than that found for the unground material. This finer material represents fresh surface produced at the expense of almost zero grinding energy. However, it is present only in small quantities at zero revolutions; therefore, it may be assumed that the effective donation of larger material to size \( x \) is completely from the 590-1190 coal fraction.
Equation (28) can then be expressed as

\[ \frac{\partial^2 p}{\partial \mu \partial R} = \pi_{590-1190} x^{k_{590-1190}} - \pi_x \left( \frac{\partial p}{\partial \mu} \right)_x \]

(35)

where the values of \( \pi_x \) are found from Figure 16. Rearranging

\[ \pi_{590-1190} x^{k_{590-1190}} = \frac{\partial^2 p}{\partial \mu \partial R} + \pi_x \left( \frac{\partial p}{\partial \mu} \right)_x \]

(36)

The second term on the right hand side of equation (36) is negligible except for \( x \) from 600 to 200 microns. Using the values of \( \frac{\partial^2 p}{\partial \mu \partial R} \) and \( \frac{\partial p}{\partial \mu} \) extrapolated to zero revolutions, the values of \( \pi_{590-1190} x^{k_{590-1190}} \) were calculated. Figure 17 shows the plot of these values for values of \( x \) from 10 to 600 microns.

From equation (24),

\[ q = \int_{x=0}^{x=M} x^{k_{590-1190}} d\mu \]

(37)

Therefore, the integrated area under the curve in Figure 17 gives \( \pi_{590-1190} \) \( q_\mu \), where \( q_\mu \) is the fraction of the material below size \( \mu \) produced by breakage of the 16 x 30 mesh sample. Taking this sample as being equivalent to coal of 900 microns, \( \pi_{590-1190} \approx \pi_{900} \), \( q_\mu \) can be estimated.

Figure 18 shows the value of \( q \) as a function of sieve size. This curve represents the breakage function of the coal of mean size of 900 microns. Although somewhat dubious assumptions have been made in the derivation of the function shown, it has the required form to lead to the overall size distributions found in the grindability tests.

Figure 18 also gives the breakage function proposed by Callcott (20).

It is clear that Callcott's function is a poor approximation in the case of the Hardgrove machine. This discrepancy cannot be explained by assuming the distribution found is the result of several consecutive breakage steps of Callcott's function, since it is clear that rebreakage...
VALUES OF $K_{590-1190}$ VERSUS SIEVE SIZE $\chi$ FOR COAL B-19447

Figure 17
BREAKAGE FUNCTION FOR COAL PARTICLES (B-19447)

OF ARITHMETIC MEAN SIZE 900 MICRONS (16 × 30 MESH)

Figure 18
of Callcott's function would make it depart even more from the distribution found. Figures (19) and (20) show the distribution plotted on probability paper, and also according to the Rosin-Rammler Law (23)

\[ R = 100e^{-\beta \mu^m} \]  

(38)

where \( R \) is the per cent by weight remaining on sieve size \( \mu \), and \( \beta \) and \( m \) are constants. Clearly the distribution does not fit either of these mathematical functions. The Rosin-Rammler plot is a straight line up to 200 microns, but over this portion of the curve, the distribution is defined as accurately by the much simpler equation (22). Consequently there seems to be no advantage in using the awkward Rosin-Rammler function.

The distribution shown in Figure 18 has the big disadvantage that it is a summation of breakage of material of size 590 to 1190 microns sieve size, whereas true breakage functions are for a fixed size. More work must be done before true breakage functions over a complete size range are obtained. No attempt has yet been made to fit the distribution to a mathematical function, except for the cases given above.

C. Grindability, Shape Factors, and Rank of Coal.

When comparing the strength of coal defined in terms of grindability with the chemical properties of coal, there is no doubt that the parameters to be used are those of ring condensation index and aromaticity proposed by Van Krevelen and Schuyer (33). However, the densities and chemical composition of the coal substance are not known at present because of the mineral matter present. Therefore the parameters cannot be determined with sufficient accuracy to allow their use. The percentage carbon content of the coal substance is not a reliable guide to its rank.
Figure 19

BREAKAGE FUNCTION OF FIG. 18 ON A LOG NORMAL BASIS

% Weight Below Sieve Size

Log Sieve Size, Microns
ROSIN-RAMMLER PLOT
FOR BREAKAGE FUNCTION OF FIG.18

Figure 20
if the coal consists of a mixture of macerals. The percentage of hydrogen is an important variable in this case. This can be seen from Figure 21 in which the carbon and hydrogen contents (Parr's basis) of a wide range of British coals (15) are plotted with their volatile matter contents. Over most of the coal range, a change of 1% in the hydrogen content has an equivalent effect on the volatile matter of about an 8.5% decrease in carbon content. Therefore, it is proposed that the rank of a coal be expressed by

\[ \text{Rank Index} = \% \text{carbon content (Parr's basis)} - 8.5\% \text{hydrogen content (Parr's basis)} \] (39)

When the volatile matter content of coals are plotted against this rank index, the points show a considerably reduced scatter about the mean line than when percentage carbon content alone is used. This is also true when grindability indices are plotted against the proposed rank index, as in Figure 22. It is clear that the grindability characteristics of a coal are closely allied to its rank. Although most of the coals plotted are British coals, which, because they are of one geological era, might be expected to form a fairly consistent pattern, the coals used in our experiments fit the mean line with as good an accuracy as the British coals. Deviations from the mean line are quite considerable in some instances, more than would be expected by experimental error of determination of C, H, or Hardgrove index, and this may be due to several causes:

a) The mineral matter of a coal might considerably influence its grindability.
Correlation of Carbon, Hydrogen, and Volatile Matter Content of Coals

Figure 21

% Carbon in Coal (Parr's Basis)

% Hydrogen in Coal (Parr's Basis)

Numbers are determined % V. M.

Key:
- Low Rank Lignite
- Sub-bituminous A
- Bituminous A
- British Coals
- U. S. Coals
- Montana lignites
- Numbers are determined % V. M.
Grindability Index and Shape Factor as a Function of Coal Rank

Rank Index = % Carbon - 8.5% Hydrogen (Parr's Basis)
b) Grindability, as measured by the Hardgrove Index, might not be an accurate representation of the grinding strength of the coal.

c) Differences in the amounts of macerals present in the coal might cause considerable change in strength.

d) The grindability might be influenced by factors which do not depend closely on rank, for example flaw structure.

At the moment, it is only possible to discuss cause (b) with knowledge obtained from our own results. The Hardgrove Index is defined by

\[
\text{Hardgrove Index} = 13 + 3.465 p_{200}
\]  

where \(p_{200}\) is the \% by weight less than 200 mesh. Figure 23 shows this \% plotted against surface area for varying revolutions for coals B-19447 and B-17790. Clearly the increase in surface area is not related to \(p_{200}\) in the form of equation (3). The surface area is not linearly proportional to \(p\) for coal B-17790, although a straight line could be drawn with a fair degree of accuracy.

Figure 23 shows \(p_{200}\) plotted against increase in surface area for the four coals ground for 60 standard revolutions, and it also shows the Hardgrove Index as a function of surface area increase. It can be seen that the Hardgrove Index is not proportional to the increase in surface area. It should be emphasized at this point that the surface areas used are those calculated on the assumptions that the shape factor is constant over the range 1 to 1190 microns and that 1 micron is an effective lower limit, whereas it has only been shown that the shape factor is constant over the range of 40 to 600 microns.
RELATION OF SURFACE AREA TO PERCENT WEIGHT THROUGH 200 MESH
FOR VARYING REVOLUTIONS OF GRINDING IN THE HARDGROVE MACHINE

Figure 23
INCREASE IN SURFACE AREA ON GRINDING
ACCORDING TO STANDARD HARDGROVE TEST
AS A FUNCTION OF % WEIGHT THROUGH 200 MESH

Figure 24
From Figure 24, it can be seen that the increase in surface area is proportional to $p_{200}$ only to a degree of accuracy of about ± 10%. This is of the same order as the deviations of Hardgrove Index values from the best fit curve in Figure 22, and it is possible that many of these deviations are caused by the Hardgrove Index (which depends on $p_{200}$) not being an accurate representation of the increase in surface area. This is particularly likely to be true where a coal fractures to give products with an abnormal shape factor, for in this case two coals might have very similar size distributions on grinding but would have considerably different surface areas. It would appear that $p_{200}$ is a better index of grindability than the Hardgrove Index both for the reasons given by Calcott (see Section III) and because it is a better index of fresh surface area production on grinding. The Hardgrove Index may be used instead of $p_{200}$, if it is borne in mind that a Hardgrove Index of 13 represents zero production of fresh surface. For scientific work it is recommended that the index used should be the increase in surface area per revolution (over the range in which linearity is obtained). There is clearly a need for the measurement of shape factors and surface area changes on grinding for a large number of coals of varying rank.

Figure 22 also shows the shape factor $k$ as a function of rank. Although definite conclusions cannot be drawn from four results, it seems probable that the variation in shape factor has a similar relation to the rank of the coals as found for grindability indices. It is unlikely that the shape factor is a function of degree of grinding; for if this were true, one would expect it to vary for different sized fractions, whereas
it is constant for a given coal over the size ranges investigated. The hydrodynamic surface area obtained with the liquid permeability apparatus should not be a function of the chemical nature of the particle surface, since the resistance to flow is due to the internal friction of the liquid; and the mean free path of the liquid molecules is not great enough compared to the flow paths for the phenomena of slip to be observed.

The variation of shape factor with rank implies that coals fracture to different mean shapes depending on their rank. A high shape factor means that the average particle is flaky, while shape factors approaching the value of six imply that the particles tend towards spherical or cubical shapes. Thus, on the basis of our limited results, anthracites and low-rank sub-bituminous coals have more flaky particles, while the easily broken bituminous coals tend to have more rectangular shaped breakage products. It is probable that shape factors also depend to a considerable extent on the petrographic constituents of the coals, since it is known that different macerals have conchoidal, splintery, or irregular breakage (36).

VIII. CONCLUSIONS

It must be realized that this is in the nature of an interim report, and, therefore, the work described contains results which are as yet incomplete. Therefore, the conclusions offered are in some cases tentative only.

1) The hydrodynamic surface area to volume shape factor was found to be constant for a given coal over the range 40 to 600 microns sieve size.
2) The ratio of the microscope projected area diameter to the nominal sieve size was found to be constant for the four coals tested over the range 44 to 88 microns sieve size. The value found for the ratio was 1.68.

3) From one test, it would appear that the log p/log μ straight line portions of the distributions found for coals ground according to the Hardgrove test can be extrapolated to at least 3 microns sieve size. The weight loss on grinding appears to be mainly material of less than 3 microns sieve size.

4) For the two coals tested at varying revolutions of grinding, a negligible amount of grinding produced about 0.4 sq. meter/100 gms. of fresh surface; but after this initial abnormal increase, the increase in surface area was proportional to the revolutions of grinding up to the condition of at least 20% of the material through a 200 mesh sieve. This was true when a lower limit of size of about 1 micron was used to calculate the surface area.

5) For coal B-19447 (a low rank sub-bituminous coal), the probability of breakage of any given size in the Hardgrove machine was found to be proportional to the size approximately to the 0.9 power. Consequently, larger sizes have a greater probability of breakage than smaller sizes.

6) An approximate breakage function for coal of mean size 900 microns was determined for coal B-19447. The function does not agree with that proposed by Callcott, nor does it fit a logarithmic/normal or Rosin-Rammler type of distribution.

7) The selection and breakage functions for coal particles of a given
size were found to change as the material was ground. A start has been made toward the accurate description of the size distribution of the product obtained after any degree of grinding from a given feed-size distribution.

8) To a crude degree of accuracy, the increase in surface area on grinding was found to be proportional to the per cent by weight passing a 200 mesh sieve.

9) As expected, the grindability indices of the coals tested were found to be related to the rank of the coals. It seems likely that the shape factors of the coals are also related to coal rank, and they were found to vary from 7.2 for a medium rank low volatile bituminous coal to 9.4 for a high rank anthracite and 9.6 for a low rank, sub-bituminous coal. It is not yet known whether two coals of similar rank may have widely different shape factors or not.

IX. FUTURE WORK

The laboratory work which remains to be done can be classified into four main sections as follows:

A. Measurements on Sub-sieve Fractions:

No experimental determination of surface areas and shape factors of the sub-sieve ground coals have been made yet because the material is too heterogenous in size to permit immediate accurate measurement. However, by repeated sedimentation into several more homogenous fractions it will be possible to determine shape factors in the manner used for the sieve
fractions. The weight size distribution of coal below 325 mesh has only been determined for one coal, and it was difficult to make definite conclusions because of the relatively large amount of fine material lost on handling. In the future, the sub-sieve fraction will be sieved out in one operation, thus reducing weight loss to a minimum and the weight size distribution determined on this sub-sieve fraction. It is hoped that a column sedimentation technique can be developed which will give the distribution accurately and rapidly without the necessity for laborious sedimentation fractionating and microscope counting.

B. Extension of Existing Results:

The results obtained so far will be repeated on a more complete range of coals and the relations of increase of surface area per revolution of grinding and shape factor with the rank of coal determined for a number of coals. It should be noted that having shown shape factors to be constant for a given coal over a wide sieve range, it will be possible to measure shape factors of new coals with much less work than required for the complete method used in this report. Results for the microscope diameter to sieve size ratio will be extended to higher sieve sizes and both of these dimensions related to the Stoke's diameter for coals of different shape factors. The validity of Rittinger's Law will be investigated using an increased number of revolutions to test whether it breaks down with the accumulation of large amounts of fines.

C. Effect of Mineral Matter:

Results can not be expected to form a coherent pattern until the influence of mineral matter is elucidated. This problem will be tackled
by differential gravity settling of coals to obtain fractions enriched or depleted in mineral matter content. A small mill will be used to measure grindability and the results correlated with those from the Hardgrove machine.

D. Breakage and Selection Mechanisms:

The information obtained from the tests described in this report was insufficient to describe the grinding mechanisms. Further information will be obtained by varying the initial size of the sample ground in the mill and also by sieving out close size fractions at a certain stage in the grinding process and regrinding this material in the absence of other sizes. Such experiments are going to require much time and effort, but it is possible that they might lead to a complete description of the grinding process in a Hardgrove Test mill. When breakage characteristics are evaluated, they may be correlated with the chemical nature of the coal, its pore structure and hardness, and its petrographic analysis. An additional approach is to vary the load on the mill and determine the effect of this on breakage products. Also the addition of lubricating agents to determine the loss of energy in frictional slip should be investigated.

It should be possible to use the techniques described and some of the results obtained in laboratory experiments, to make similar studies on industrial grinding equipment, with the view of determining optimum operating and design conditions for specified performance.
X. ACKNOWLEDGMENTS

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DERIVATION OF PERMEABILITY FORMULA

Consider Figure A1. Let \( L \) be the length of the packed bed and let \( \varepsilon \) be its porosity.

For streamline flow in a cylindrical pore, Poiseuille's Law states that

\[
V' = \frac{\pi}{8} \frac{r^4}{\eta} \frac{dp}{dx}
\]  

(A1)

where \( V' \) is the volume flow, \( \frac{dp}{dx} \) is the pressure gradient, and \( \eta \) is the absolute viscosity of the fluid flowing. Considering the bed as a conglomerate of tortuous pores, it is clear that the total volume flow along the respective axes of the pores is greater than that along the axis of the bed. This difficulty is more conveniently overcome by considering the velocity of flow along the pores. Then (A1) can be re-expressed as

\[
U_{\text{eff}} = \frac{\pi}{8} \frac{r^4}{\eta} \frac{1}{\pi r^2} \frac{dp}{dx} = \frac{r^2}{8\eta} \frac{dp}{dx}
\]  

(A2)

Let the true length of the pores in the bed be \( L_e \) and let \( \frac{L_e}{L} = \varphi \) (tortuosity factor). The resolved overall fluid velocity along the axis of the bed is then, from the Dupuit relation,

\[
u = \varepsilon \frac{U_{\text{eff}}}{\varphi}
\]  

(A3)

Substituting (A3) in (A2) to eliminate \( U_{\text{eff}} \),
DIAGRAM OF PERMEABILITY APPARATUS

Figure A1
\[ u = \frac{\varepsilon}{q} \frac{1}{8 \eta} r^2 \frac{d\rho}{dx} \]

where \( \frac{d\rho}{dx} \) is still the gradient along the pore axis.

If \( \frac{dV}{dt} \) is the total volume flow rate through the bed of radius \( R \),

\[ \frac{dV}{dt} = \pi R^2 u \]

Therefore,

\[ \frac{dV}{dt} = \frac{\pi R^2 \varepsilon r^2}{8 \eta} \frac{d\rho}{dx} \]

For a liquid the volume and viscosity are independent of pressure, therefore

\[ \frac{dV}{dt} = \left( \frac{\pi R^2 \varepsilon r^2}{8 \eta} \right) \frac{\Delta p}{q} \quad (A4) \]

where \( \Delta p \) is the pressure differential across the bed, along the axis of the bed, at time \( t \). Consider a small volume flow \( dV \) which leads to the height of fluid in the right hand side going from \( h_1 \) to \( h'_1 \) and that of the left hand side going from \( h_2 \) to \( h'_2 \), then

\[ dV = \pi R_2^2 (h_2 - h'_2) = \pi R_1^2 (h'_1 - h_1) \]

That is,

\[ \frac{dV}{\pi R_2^2} + \frac{dV}{\pi R_1^2} = (h_1 - h'_1) + (h_2 - h'_2) = (h_1 - h_2) - (h'_1 - h'_2) \]

or

\[ \frac{dV}{\pi} \left( \frac{1}{R_2^2} + \frac{1}{R_1^2} \right) = d\ell \quad (A5) \]
where $dl$ is the differential head across the bed. If $l$ is in cms. of mercury, and other units are c.g.s., then \( \Delta p = \frac{l \times 10^6}{75} \) dynes cm.$\textsuperscript{-2}$. and substituting (A5) in (A4)

$$\frac{dl}{dt} = R^2 \left( \frac{1}{R_2^2} + \frac{1}{R_1^2} \right) \cdot \frac{\varepsilon r^2}{8 \eta q^2 L} \cdot \frac{10^6}{75} \cdot l \quad (A6)$$

or

$$\frac{dl}{dt} = A \cdot l \quad (A6a)$$

where $A$ is constant for definite experimental conditions. Integrating between $t = 0$, $l = l_1$ and $t = t$, $l = l_2$,

$$\log_{10} \frac{l_2}{l_1} = \frac{\varepsilon r^2}{\eta q^2 L} \cdot \frac{10^6}{1384} \cdot t - R^2 \left( \frac{1}{R_2^2} + \frac{1}{R_1^2} \right) \quad (A7)$$

When $R_2 \gg R_1$, as in the case of our apparatus,

$$\log_{10} \frac{l_2}{l_1} = \left( \frac{R}{R_1} \right)^2 \frac{\varepsilon r^2}{\eta q^2 L} \cdot \frac{10^6}{1384} \quad (A7a)$$

If $S_o$ is the specific envelope surface area of solid per cm.$^3$ of solid,

$$m = \frac{\varepsilon}{S_o (1 - \varepsilon)} \quad (A8)$$

where $m$ is the mean hydraulic radius. Let $k_o$ be defined by

$$(2m)^2 = \frac{k_o r^2}{2} \quad (A9)$$

Then

$$\log_{10} \frac{l_2}{l_1} = \left( \frac{R}{R_1} \right)^2 \frac{2 \varepsilon^3 t}{S_o^2 (1 - \varepsilon)^2 \eta q^2 k_o L} \cdot 346$$

That is,

$$S_o = \frac{\varepsilon t}{m L \log \left( \frac{l_2}{l_1} \right)} \quad (A10)$$
For the spherical equivalent diameter \( d \),

\[
d = \frac{6}{s_0} \text{ cm}
\]

Therefore,

\[
d = \left( \frac{k_0 q^2}{4} \right)^{\frac{1}{2}} \left( \frac{R}{R} (1 - \varepsilon) \right) \sqrt{\frac{\eta L \log(\frac{1}{\varepsilon})}{\varepsilon t}} \tag{A11}
\]

Equation (A11) is only of use where \( k_0 q^2 \) has a definite value which is determined by calibration. Where a bed occurs which is floculent, the bed becomes non-homogenous, bridging occurs, and \( q^{-} \) is variable depending on how the bed is packed. For consolidated densely packed beds, it has been found that the effective value of \( q^{-} \) may reach very high values (80 or 100 instead of the \( \sqrt{2} \) expected). This is because the flow equations are only true when the constrictions between voids are almost as large as the voids themselves. It is obvious that these constrictions approach zero radius \( u \) becomes small, although \( \varepsilon \), \( q^{-} \), and \( S \) may not be much different from another system for which the relations hold. As \( u \) becomes small, \( q^{-} \) appears to become large. Physically this means that the porosity and radius which should be used are those of the pore constrictions which are controlling flow and not those of the pores as a whole. This effect is noted when the ratio of largest to smallest particles in the bed becomes greater than about 3 to 1.