The *Cooling* of *Cakes* by Newton's Law

By Dale Miller, Engineering Science, 1st Year Student
along with Instructor Dr. Ermek Nurkhaidarov

Abstract

Newton's Law of Cooling is based on the equation dT/dt=k(M-To), where To is the temperature of the cake, dT is the change in the temperature of the cake, dt is the change in time, k is a constant, and M is the temperature of the medium which in this case is the air surrounding the cake. By baking a set of test cakes to find the value of k, and finding the slope of the second batch's cakes, I will test my hypothesis through the experiment to compare the error from both Newton's law of cooling and a linear regression/ progression.

Purpose

To observe the function of derivatives in the cooling of cakes, I wanted to bake two batches. Derivatives can be used in many calculations such as velocity, acceleration, etc. so it is important that we understand them, especially in math and physics. I hypothesize that Newton's Law will better predict the temperature of a cake after fifteen minutes whether the room temperature is higher or lower than the cake.

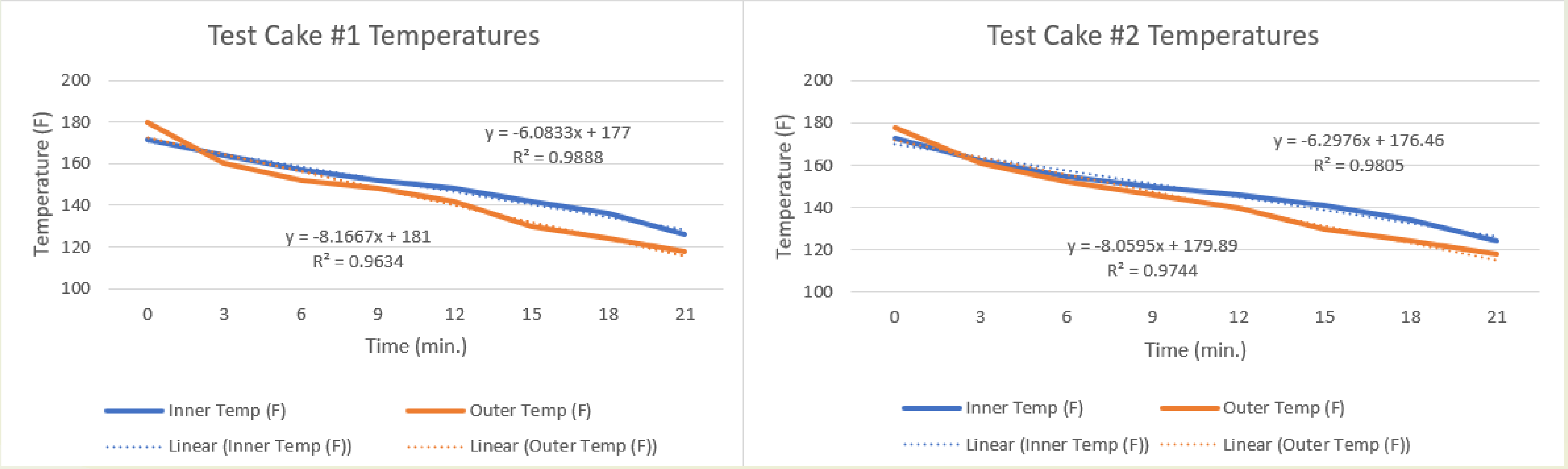
Materials

- Oven
- Cake Mixing Utensils and Bowl
- 2 Betty Crocker Super Moist cake mixes
- 2 cups of water
- 1 cup of vegetable oil
- 6 eggs
- 2 8" metal cake pan rounds
- Thermometer with a Range of at Least 0-300 Fahrenheit
- Clock/Stopwatch.

Procedure

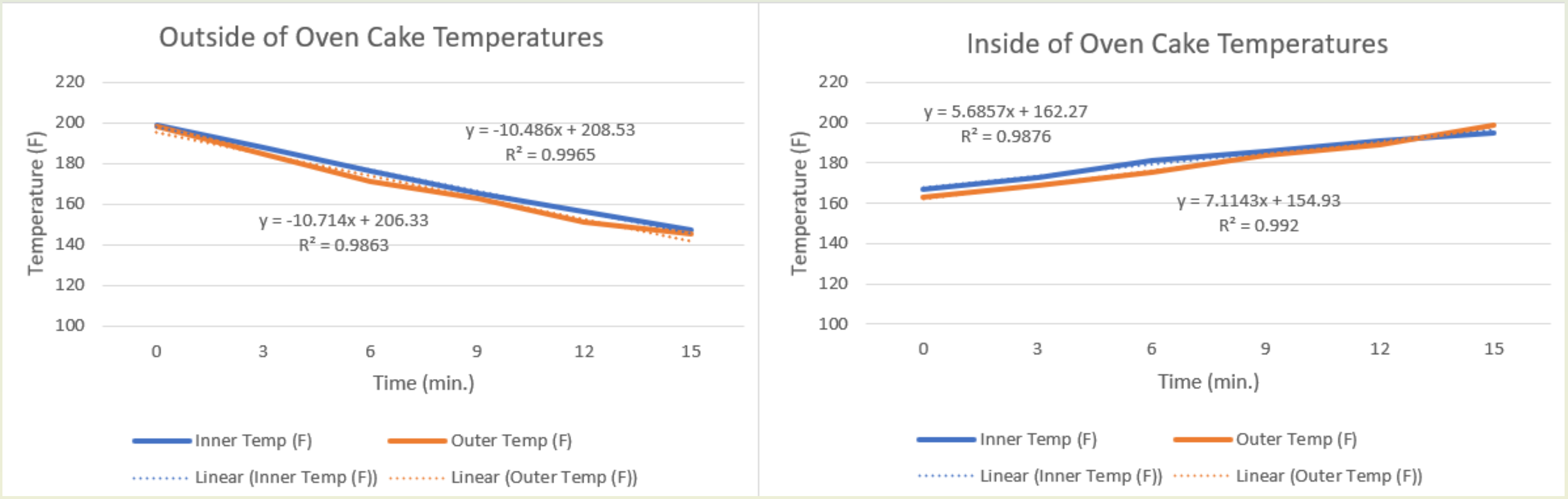
1. Make enough cake mix for two cakes from one of the Betty Crocker Super Moist cake mixes. This involves mixing 3 eggs, 1 cup of water, and ½ cup of vegetable oil with the dry cake mix.
2. Pour the wet cake mix into the 2 8" metal cake pan rounds.
3. Cook the cakes in the oven at 350 degrees Fahrenheit for 30 minutes.
4. After 30 has passed, take the cakes out of the oven. Use the thermometer and record the temperature of each cake in the middle and outer ring for each right as they come out of the oven and the temperature of the room and record the information.
5. Continue to check the temperature of the middle and outer ring of each cake and record the information every 3 minutes as the cakes cool. Continue to record the temperature for a total of 8 times over 21 minutes (initial temperature and 7 temperatures in 3-minute increments).
6. Now that you have a change in temperature, a change in time, a the value of the medium/room temperature, derive the value of the constant k by manipulating the initial formula and integrating creates TF=Ae^(-kt)+M, where A=To-M and TF is the final temperature. To get an average k, take the k when t=3 and divide by 7.
7. Now that the average k is known, predict the temperature of cake in warmer and colder temperatures. To test this, complete steps 1-4 at a lower and higher medium/room temperature. For my project, I used 50 degrees and 200 degrees F since my original temperature was 74 degrees F.
8. To compare the predicted value of Newton's law to a linear regression, use TF= at+To. During step 4, find the difference of the initial temperature and the time that has passed to find the value of the slope (a) for the linear regression. The initial temperature will be To.
9. After any given time, t, check the temperature of the cake for the higher and lower room temperatures. Compare the value found for each different room temperature with their respective TF value found through Newton's law of cooling and the linear regression to see which method is closer to the actual value from the second batch of cakes.

The First Cakes



These graphs show the change in temperature of the first two cakes that I used to figure out the value of the constant k for the inner part of the cake and the outer ring.

The Second Batch



These graphs are for a lower temperature of 50 degrees F (left) and a higher temperature of 200 degrees F (right) of the medium/room,

Results

K #1 Inner	K #1 Outer	K #2 Inner	K #2 Outer
0.028	0.046	0.039	0.040
0.027	0.024	0.028	0.026
0.016	0.016	0.017	0.021
0.017	0.022	0.018	0.023
0.028	0.041	0.024	0.036
0.025	0.027	0.031	0.027
0.057	0.030	0.059	0.030
Average		Average	
0.028	0.029	0.031	0.029

The K values for the 2 test cakes with t=3 and the average for the inner and outer ring.

From The First Test:					From the Actual Test:			
Expected Temperatures after 15 minutes (based on K)					Actual Temepratures after 15 minutes			
When M is 50		When M is 200			When M is 50		When M is 200	
Inner	Outer	Inner	Outer		Inner	Outer	Inner	Outer
147.9	145.8	178.3	176.1		147	145	192	199
Expected Temperatures after 15 minutes (Statistic Linear Regression)					Actual Temperatures after 15 minutes			
When M is 50		When M is 200			When M is 50		When M is 200	
Inner	Outer	Inner	Outer		Inner	Outer	Inner	Outer
144	133	197	193		147	145	192	199

The comparison of the predicted values based on Newton's law of cooling and a linear regression and the actual values from the second batch of cakes.

Analysis

The predicted value based on Newton's law of cooling and the constant K is very close for when M=50 degrees F with an error of less than 1% for both the inner and outer ring. Whenever M=200 degrees F, the error is 7% and 11% for the inner and outer ring, respectively.

The predicted value based on the linear regression when M=50 has 2% and 8.2% error for the inner and outer ring, respectively. When M=200 degrees F, the error is much smaller at 2.6% and 3% for the inner and outer ring, respectively.

Both methods have a similar amount of error for both the inner and outer ring of the cakes, with error that is within 10% of the opposite ring.

When comparing the two methods, Newton's law of cooling was much closer to the actual temperature of the cake whenever the medium was at a lower temperature than the cake, as shown by when the M=50 degrees F. However, the linear regression was superior to predicting the temperature of the whenever the room temperature was higher than the cake.

Conclusion

Overall, the data really speaks for itself in how Newton's law of cooling was superior to a linear regression. Whenever the value of the room temperature, or M, was lower than the temperature of the cake, I was able to predict with less than 1% error the temperature of the cake after 15 minutes. While the error of the temperature found by Newton's law is higher than a linear progression whenever the room temperature was higher than the cake, I would like to contribute this to a fluke. A linear regression in a longer time period would not work because the temperature of the cake would climb above the room temperature, which is not realistic. In the future, I would like to do a longer test for when the room temperature is higher than the cake's temperature to prove this theory. My hypothesis, however, is only mildly correct for the data that I collected during the experiment since the error for newton's law of cooling was lower for only one of the cakes.