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Modeling the Flexure of the Mediterranean in Response to the End of the Last Glacial Maximum

A Senior Thesis in Geosciences

by

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Abstract

Stresses from surface loads, even when unrelated to plate tectonics, may influence deformational processes. Specifically, McGuire et al. (2000) suggested that, in the Mediterranean, the flexural stresses from the sea level rise following the Last Glacial Maximum may have contributed to the concurrent increase of volcanic activity. Using topography and bathymetry data from the ETOPO2 database, we use the elastic plate flexure model to calculate the spatial distribution of flexural stress from a 125m sea level rise in this region. From this, we find that the tensional stresses from SL rise are large enough to contribute to the cause of the active period of volcanism around 15-8ka, and that the modeled stress directions and magnitudes predict that normal faults in western Apennines are more active and thrust faults in eastern Apennines are slowed or deactivated, which may affect frequency of seismic events. Overall, our study supports to the development of the idea that rapid sea level rise produces a significant load that may have a considerable effect on crustal stresses and processes.
Introduction

Traditionally, tectonics has been thought to completely govern regional stress regimes. Recent studies have suggested that there may be other contributing factors, including large changes in surface loads, which produce significant changes in the magnitude of stresses without any relationship to plate tectonics. When these differential loads occur as a result of deglaciation, the changes in stress occur rapidly enough so that it is possible that they influence deformation. This idea can be applied to current state of stress and recent deformation in the Mediterranean.

Several other papers have examined the idea that large changes in load may impact deformation, and seismic and volcanic activity. For instance, Luttrell and Sandwell (2010) investigated the plate bending at coastal regions as a response to the change in load resulting from eustatic sea level change after the Last Glacial Maximum (Luttrell and Sandwell, 2010). They proposed that load changes resulting from post-glacial sea level rise promote failure on onshore transform faults, and alter the seismic cycle of secondary faults near plate boundaries (Luttrell and Sandwell, 2010). Similarly, Grollimund and Zoback (2000) studied the change in stress field in northern United States as a function of retreat of the Laurentide Ice Sheet and suggested the magnitude of the changes in seismic strain rate were large enough to trigger subsequent earthquakes (Grollimund and Zoback, 2000). Finally, a study by McGuire et. al. (1997) suggested that the stress-related influences of sea level rise and the level of explosive volcanic may be linked, which served as motivation for our research (McGuire et al., 1997).
The deglaciation following the Last Glacial Maximum caused eustatic sea level rise, which increased the weight of water affecting areas near coastlines. The rate of this sea level change (up to about 1.25 cm/ year until about 4ka) was greater than the rate of stress relaxation, which is the dissipation of stress by slow deformational processes. (Luttrell and Sandwell, 2010). In this case, the addition of water acted to increase the load on the Mediterranean lithosphere in large enough magnitudes that it may be significant enough to influence deformational processes. In this paper, we use 3D flexural modeling to attempt to quantify the bending stresses from the addition of water after the glacial period and determine the possibility of a relationship with regional volcanic activity and faulting. This model uses topography and bathymetry data from the ETOPO2 database from the National Geophysical Data Center (NOAA) along with constant value inputs to determine flexural stresses in 3-D using the equations in the elastic plate (flexure) model from Watts (2001). In this way, this study aims to increase understanding of how post-glacial sea level rise might affect crustal stresses.

**Background**

The current geology of the Mediterranean is complex due to movements of microplates, but is broadly characterized by connected fold and thrust belts and extensional basins, which are a product of the interaction between the Eurasian and African-Arabian plates, including the creation and closure of the Paleotethys and Neotethys ocean basins (Cavazza, 2003). The complexities of the collisions between the Eurasian and African-Arabian plates, resulted in a region of intricate plate/microplate arrangements, where crustal blocks move independently or partially independently from one another (Anzidei et al., 2001). Some of the geologic features
of the region, including major faults, basins, and water depths, are shown in Figure 1. The material that makes up the labeled basins varies in each region. For example, the floor of the Lingurian and Tyrrhenian Basin is composed of Neogene oceanic crust, while the shallower Adriatic basin is continental lithosphere (Cavazza, 2003).

![Figure 1- Tectonic Setting of Western and Central Mediterranean Sea (Oldow et al., 2002). Black lines: transcurrent faults; Black toothed lines: contractional faults; Blue lines: Extensional Faults; Water Depth of 0 to 1000m: White; Water depth> 1000m : Light Green.](image)

The Last Glacial Maximum peaked approximately 20,000 years ago, during the Pleistocene epoch (Grollimund and Zoback, 2000). In the Mediterranean, the LGM coincides with a large depression in moisture and precipitation, as well as estimated temperatures 8-9°C below current temperatures(Hughes et al., 2007). As the climate warmed following the glacial period, the sea level rose about 125 meters due to melting ice sheets across North and South America, Eurasia, Greenland, and Antarctica (Poore, 2000). The stresses from this sea level rise are the central focus of this study.
Computational Methods

The MATLAB model developed and used in this study is based on the elastic plate model (also known as the plate flexure model), which was described by Jeffreys, Meinesz, Gunn, and Walcott (Watts, 2001). This theory describes the behavior of the crust and upper mantle during loading, with the main assumption that both behave as perfectly elastic materials, meaning stresses during bending are linearly proportional to strain. While the assumptions in this model greatly simplify flexural calculations, it is still widely used in geology for applications including glacio-isostatic rebound, rapid seamount growth, and accumulation of river delta sediments, because it is useful as a standard for comparison of real-world observations (Watts, 2001)

The ETOPO2 database of the National Geophysical Data Center within the National Oceanic and Atmospheric Administration served as the main source of data for the spatial geometry of the water load. It provides bathymetry and elevation data to a user-specified area with a 2-minute cell size (NOAA, 2012). The source data are downloaded for an area larger than the study area, but later cropped within the model to eliminate the influence of edges with null values.

The starting point for the flexural model is to define the flexural rigidity (a measure of the bending moment acting on the per unit length per unit curvature) of the plate:

\[ D = \frac{E \cdot T e^3}{12(1 - v^2)}, \]  

(1)
where $D$ is the flexural rigidity, which is a function of the elastic thickness, $Te$, Young’s Modulus $E$, and Poisson’s Ratio $v$. In this paper, the elastic thickness is defined as “the thickness of an elastic layer that would respond to applied loads in the same way as the heterogeneous lithospheric plate” (Tesauro et. al., 2011). The elastic thickness varies within the study area from roughly 15 to 35 km (Tesauro, 2011) (Watts, 2001). Young’s modulus is the ratio of tension or compression to extension in a solid that is under axial compression or tension and is unrestricted laterally, and is estimated to be about 56e10 Pa. Finally, Poisson’s ratio is the ratio of lateral extension to longitudinal extension in the same conditions. In this study, we set Poisson’s ratio equal to a constant value of 0.25. Since Young’s Modulus, Poisson’s Ratio, and elastic thickness are all treated as constant, the flexural rigidity is constant over the entire model area.

The flexural rigidity is used in elastic plate response to calculate the flexural response function $\phi$, which influences the spatial variation in flexure due to a load on an elastic plate. The flexure can be calculated using an adaptation of the equations for the flexural response function and for the Fourier transform of the flexure, $Y(k)$, respectively:

$$\phi = \left[ \frac{D \cdot k^4}{(\rho_m - \rho_w) \cdot g + 1} \right]^{-1}$$  \hspace{1cm} (2)

$$Y(k) = -H(k) \frac{\rho_c - \rho_m}{\rho_m - \rho_c} \phi(k)$$  \hspace{1cm} (3)

where $\rho_m$ is the density of the mantle (3300 kg/m$^3$), $\rho_w$ is the sea water (infill) density (1030 kg/m$^3$), $\rho_c$ is the density of the crust, $H(k)$ is the Fourier transform of the of the topography, $g$ is
gravitational acceleration (9.81 m/s), and $k$ is the wave number of the load in the x direction (a long wavelength variation corresponds to a small wave number, and a large wave number corresponds to a short wavelength variation). In this case, $k$ is divided into $k_x$ and $k_y$; the wavenumbers in the x and y directions. These equations allow us to determine the flexure in 2-D as a response to the addition of the load, in this case, the addition of water due to the sea level rise on the Mediterranean following the Pleistocene glacial period.

The flexural stress, $\sigma$, resulting from the bending caused by the vertical load is calculated using:

$$\sigma = \frac{1e^{-6} \cdot E \cdot Yf}{1 - v^2} \frac{d^2y}{dx^2},$$

where $Yf$ is the vertical distance in meters from the neutral surface at which the flexural stress is calculated and $d^2y/dx^2$ is the second derivative, or curvature, of the flexure. Figure 2 shows an example of the derivatives of flexure with a simple water loading. As curvature increases, the flexural stress increases. This equation allows us to determine if the magnitude of stresses from the load are significant enough to possibly have an impact on deformational processes.

The flexure and flexural stress are plotted in several different forms, and are then used for making inferences about the correlation of flexural stress due to post-glacial sea level rise with the current stress regime and historical volcanic activity. Post-Pleistocene volcanic data used for comparison is largely from research based on deep sea sediment cores which records volcanic activity for the past 110kyr (McGuire et al., 1997). Data specifically for the comparison with Etna was based on chemical analysis of pyroclastic products in various tephra layers (Coltelli et al., 2000).
Figure 2- Diagram of flexural stress derivatives and distributions after the addition of water. The finite neutral surface (where stress is zero) is marked. Compressional stresses are shown in blue with converging arrows while extensional stresses are red with diverging arrows.

Results

This model was tested for sensitivity to the input cell size, meshgrid size, and several of the unknown and estimated parameters. The topographic data were downloaded from the ETOPO2 database in 2 forms: a 2-minute grid and a 1-minute grid. Running the program showed that the differences between the two were negligible, except in run time, so the 2-minute grid data was used. A smaller meshgrid increases the spatial resolution, but lengthens the run time. A
compromise between the two resulted in matrices that are 1024x1024 in size (equal to about a 2x4km cell size), except Figure 9, for which the matrix size is 256x256 (~10x16km cell size).

The sensitivity analysis also revealed that the model is not very sensitive to the density of water or density of the mantle. However, this model is very sensitive to the elastic thickness value, in that greater thickness gives larger stresses. Based on Tesauro et. al. (2011), the elastic thickness in the Mediterranean varies from approximately 15 to 35 km. So, for the results shown below, an elastic thickness of 20km was used to avoid exaggerating the values of these stresses.

As mentioned, the flexural model calculates results based on data from the ETOPO2 database. Our model initially plots this regional topography and bathymetry data, as is shown in Figure 3.

![Figure 3- Topography and Bathymetry data from ETOPO2 database (2 minute grid) used as input for the model. Black line indicates the coastline.](image)
The spatial geometry of the load (Fig. 4) varies according to the regional bathymetry. Anywhere the elevation value for the topographic data is above sea level (>0m) is assigned a value of 0. Anywhere the elevation value is below sea level (<0m) is considered to be part of the load. During this time period, the sea level in the Mediterranean is expected to rise by 125 meters (Poore, 2000). For this reason, anywhere the depth is greater than 125, the height of the load is reduced to 125 meters. Overall, the shape of the load mimics the current Mediterranean Sea, but with a maximum height of approximately 125 meters.

![Spatial Geometry of Load (m)](image)

**Figure 4** - Load geometry of water added to the plate. Calculated based on the bathymetry data. Dark red areas experienced no surface loading from sea level rise and areas of blue represent the largest height of water load (125m).

When this load is applied to the Mediterranean region, our model predicts the lithosphere flexes as shown in Figures 5 and 6. The greatest magnitude of downward (negative) flexure is found to be just offshore. Similarly, the most upward (positive) flexure is concentrated along the coastlines, especially in the Italian peninsula and Sardinia. The values of these flexures range from a few meters of uplift to approximately 40 meters of flexural subsidence.
Figure 5- 3D flexure model of plate flexure from the addition of the load, given an elastic thickness of 20km. The colors indicate the magnitude and direction of flexure in meters. Negative values correspond to a downward flexure and positive values correspond to upward flexure.

Figure 6- Map view of plate flexure from the addition of the load, given an elastic thickness of 20km. The colors indicate the magnitude and direction of flexure in meters. Negative values correspond to a downward flexure and positive values correspond to upward flexure.
Finally, the model calculates the flexural stress at the top of the plate, at a defined “stress depth” and as a sliced 3-d cube of data. The calculated values for flexural stress in the region are significant, ranging from approximately 1 MPa to -1 MPa at the top of the plate. The red, positive regions of flexural stress are indicative of tension, whereas the blue, negative regions of flexural stress represent compression. The flexural stress magnitude (Figures 7 and 8) is largest in Southern Italy, Corsica, Sardinia, Sicily, and the Adriatic Sea.

Figure 7- Flexural Stress at the top of the plate from addition of load. Tensional areas have positive (red) values and compressional areas have negative (blue) values.
Figure 8- Flexural Stresses at top of plate from addition of load and location of principle volcanic centers active during the late Quaternary. Volcanic centers modified from McGuire et. al. 1994. Tensional areas have positive (red) values and compressional areas have negative (blue) values.

A cross section of stress at the approximate location of Etna (Figure 9) reveals how the flexural stresses change with depth. The finite neutral surface, where all stresses have decreased to zero, is located at 10km below the surface, because the elastic thickness was assigned a value of 20 km. Beneath the finite neutral surface, the sign of the stresses is reversed, and the magnitudes increase again. The flexural stress is larger at great depths, changing the range of values to approximately +/- 2MPa.
Discussion

In general, the model results show that the magnitudes of flexural stresses, reaching up to approximately 1.5 MPa are large enough to impact deformational processes. This magnitude of stress is well above the minimum needed to be significant in other tectonically active regions. For example, the results of King et. al. (1994) reveal that aftershocks of the Landers earthquake most commonly occur along faults with a Coulomb stress change of as low as one-half bar (0.05 MPa) (King, 1994). King, Stein, and others invoke the theory of self-organized criticality to explain the correlation between local earthquake triggering and surprisingly small stress changes. This theory states that large natural systems organize themselves so that they are poised at a critical state — right at the threshold. In this case, it would suggest that the brittle crust in tectonically active areas is at the verge of failure, so that seemingly small perturbations
in stress can have a large (or chain) reaction, and earthquakes could be triggered distant or near to the original event (King, 1994). Similarly, Luttrell et al. (2007) studied the effect of lake loading along the San Andreas Fault, with flexural stresses of 0.1-0.2 MPa, and concluded that, when faults are near critical stress, this small change in surface loading may influence earthquake cycling (Luttrell et al., 2007).

The average magnitudes of stress of +/-1MPa (+/-10 bars) in our model are much larger than those used to explain the spatial arrangement of aftershocks, so this same theory of self-organized criticality may apply to our study. Since the crust in the tectonically active Mediterranean region is theorized to be in a critical state, the flexural stresses calculated here should be large enough to influence deformation.

The model also supports the preliminary conclusion of correlation between sea level and frequency of explosive volcanism by McGuire et al. (1997), who used tephra layers to identify exceptionally intense periods of Mediterranean volcanism in the past 100 ka. They suggest that the unusual period of increased volcanic activity 15 – 8 ka, which was the most intense episode of the past 100 ka, was a result of the rapid sea level rise at the end of the last glacial maximum (McGuire et al., 1997).

Both compressional and tensional stresses can have an effect on magma chambers in order to influence volcanic activity. Compressional stresses in the crust can apply pressure to a magma chamber, pushing magma upward. On the contrary, a decrease in compressional stress (or an increase in tensional stresses) can promote melting, bubbling, and increase the flow of magma.
through conduits (Hill, 2002). We predict that the tensional stresses on the brittle rocks on the volcanic areas in the Mediterranean ease the rate of flow of magma, thereby contributing to a period of increased volcanic activity. Our stress modeling supports this finding by validating that the magnitude of flexural stresses from the post-glacial sea level rise is large enough to influence volcanic activity.

In general, the responsible volcano cannot always be identified for each tephra layer, and the behavior of individual volcanoes can be affected by more small-scale influences. So, analyzing the eruptive history from a single volcano should not always reproduce this same period of increased explosive volcanism. The volcanic history of some of the major areas of volcanism (Figure 8) was investigated, to see if the documented periods of increased volcanism coincided with the 15-8 ka period of increased volcanism in the Mediterranean.

The recent eruptive history of Etna correlates well with the timing of oceanic loading. Approximately 100ka, stratigraphic correlation between a lava flow and a 147 and 107ka lava sequence indicate that Etna may have had some strombolian and subplinian activity (Coltelli et al., 2000). Between 100 and 80ka, pyroclastic deposits from Etna indicate plinian benmoreitic activity (Coltelli et al., 2000). The record from 80 to 16ka is discontinuous, but Etna is considered to have experienced a period of relative quiescence approximately 22 to 15.5 ka (Coltelli et al., 2000; McGuire et al., 1997). Following this period of inactivity, a phase approximately 15.5-15 ka was identified by Coltelli et. al. as intensely explosive (2000). Similarly, the one ash layer attributed to Etna by Paterne et. al. was dated to be approximately 14.18ka. Finally, there is a period of eruptive activity about 13ka with basaltic activity of strombolian and subplinian type.
According to our model, the area around of Etna was subjected to approximately 1MPa of tensional stress (Figures 7, 8, and 9) as a result of this flexure. The period of inactivity from 22 to 15.5ka coincides with the low sea level during the Last Glacial Maximum. Then, the plinian eruptions of 15.5-15ka could be influenced by the extensional stresses related to post-glacial sea level rise. Assuming the relaxation time is on the order of $10^4$ years, the aforementioned period of eruptive activity about 13 ka, may also be related to the post-glacial sea level rise stresses (Coltelli et al., 2000).

In other areas of tensional stress predicted by our model, other volcanoes show a rise of activity around 15-13 ka as well. The area of Campi Flegrei is documented as having an increase in volcanic activity over the past 16 ka, after about 8,000 years of relative quiescence (Paterne et al., 1988). However, it is hard to attribute this rise in activity to the flexural stresses due to sea level rise in the Mediterranean, because it is correlated with a decrease in Ischia activity, which would be subject to roughly the same flexural stresses. Similarly, Vulcano, part of the Eolian island arc, is hypothesized to be responsible for 2 ash layers dating 13.9 and 11.9ka, indicating it was active during this time period, but not necessarily uncharacteristically (Paterne et al., 1988). The Aegean arc, including Santorini, was thought to be in a long period (~15,000 years) of volcanic quiescence starting around 18ka (Vinci, 1985).

Finally, the water-loading stresses might have had an effect on tectonic processes at the time, especially in the Apennine Mountain range. The Apennine Mountain range is a complex mountain chain approximately 1500km long running through Italy. It formed from an oblique
collision between the African and Eurasian plates, influenced by multiple microplates, which occurred during the Tertiary and Quarternary. The mountain range is compressional in the east, shown by a region of thrust faulting, and extensional in the west, with normal and strike/slip faults (Figure 10) (Stanley, 1985).

Our model predicts the addition of the load would put a large part of the southern Apennine mountain range under tensional stress of up to about 0.7 MPa (red oval and blue oval in Figure 10). With this amount of stress, it is possible that there would be a period of time in which normal faults on the western part of the Apennine Mountain Range (red oval in Figure 10) are more active. The thrust faults located at the eastern part of the Apennine Mountain Range (blue oval in Figure 10) would be predicted to slow down or deactivate as a result of this tensional stress. The area of thrust faults to the east of the Italian peninsula in the Adriatic Sea (yellow
oval in Figure 10) is subject to up to 1MPa of compressional stress (Figure 7). Therefore, we would expect these thrust faults to be more active at this time. Areas of affected faulting activity from flexural stresses could relate to increased or suppressed earthquake activity. With post-glacial sea level rise cycling approximately every 100 thousand years in the Pleistocene, we would expect to see cycling behavior in faulting and earthquake activity. However, current paleoseismic data in this region is not comprehensive enough to show a relationship between these stresses and earthquake activity.

Conclusions

This mathematical model, based on elastic plate theory, attempts to compute the stresses due to sea level rise after the Pleistocene glacial period. The calculations reveal that the change in surface loading due to a 125m rise in the Mediterranean Sea produces flexural stresses at the surface around +/- 1 MPa. Based on related studies, we confirmed that this level of stress is significant enough that it might impact the state of stress in the region, and it is reasonable to investigate a correlation with an increase in volcanic activity. We suggest that the tensional stresses from sea level rise eased the flow of magma, contributing to the cause for the abnormally active period of tephra emplacement around 13ka. These stresses may have other impacts on the region, including influencing the faulting in the Apennine Mountain range. Overall, this study supports the development of the recent idea that changes in surface loads, including sea level rise, may influence deformational processes completely independently of plate tectonics by quantifying the flexural stress magnitudes, and confirming that they are large enough that they are capable of influencing geologic processes, including seismicity and volcanism.
References


Grollimund, B., and Zoback, M. D., 2000, Post glacial lithospheric flexure and induced stresses and pore pressure changes in the northern North Sea: Tectonophysics, v. 327, no. 1–2, p. 61-81.


% Govers Med Miocene Reconstruction

clear all;

load Mediter3.txt; %from the etopo database
long=Mediter3(:,1); %assign long, lat, and elevation
lat=Mediter3(:,2);
elev=Mediter3(:,3);

npts=512;

yl=lat*111; % conversion to km

x1=long.*(2*pi*6371*cosd(lat))/360;

%%
c=1; %assign counters for loop and if/and
a=1;

%for loop adjusts the boundaries of the map, to eliminate some stretching
%and "fake" zero boundaries

for c= 1:length(yl)
    if (((yl(c))>3000) && ((yl(c))<5500)&& ((x1(c))<3500) && ((x1(c))>-500)) %values to limit size
        y(a)=yl(c);
        x(a)=x1(c);
        elev(a)=elev(c);
        a=a+1;
    end
end

%%

tx=(min(x)):1:(max(x)); %x grid points min : increment : max
ty = (min(y)):1:(max(y)); %y grid points min : increment : max
gx=linspace(min(x),max(x),1024);
gy=linspace(min(y),max(y),1024);

[XI,YI]=meshgrid(gx,gy);
ZI=griddata(x,y,elev,XI,YI);

figure(2);
pcolor(gx,gy,ZI); shading interp; colorbar;
hold on;
v=[0]; % contour lines
contour(gx,gy,ZI,[v v],'k');
hold off;
xlabel('Longitude(km)')
ylabel('Latitude(km)')
title('Current Topography of Study Region(m)')

%%
% just water
ZI2=ZI;
nn=isnan(ZI2);
l=ZI2>0;
b=ZI2<-125;
ZI2(l)=0;
ZI2(b)=-125;
ZI2(nn)=0;

figure(3);pcolor(gx,gy,ZI2); shading interp; colorbar;
title('Spatial Geometry of Load (m)')
xlabel('Longitude (km)')
ylabel('Latitude (km)')

%% flexure
%==============================
% Te=20e3; % km elastic thickness

g=9.81; % gravity
v=0.25; %Poisson's ratio
E=56e10; %Young's modulus

npts=1024;

thick=35e3; % m crust thickness

rho_load = 2700;
rho_infill = 2500;
rho_water = 1030;
rho_mantle = 3300;

hk_sea = fft2(ZI2);

% Te=20e3; % km elastic thickness

ty = 2*pi/3.6e3*(0:1:512)/1024;
tx = 2*pi/8.4e3*(0:1:512)/1024;

ty = [ty flipr(ty(2:512))];
tx = [tx flipr(tx(2:512))];

%equations in watt's book
[kx,ky]=meshgrid(tx,ty);

kk = sqrt(kx.^2+ky.^2);

D=E*Te^3/(12*(1-v^2)); % flexural parameter

wk_sea = (rho_water).*(g./(kk.^4.*D+(rho_mantle)*g)).*hk_sea;
w1 = real(ifft2(wk_sea));

figure(4); surf(gx,gy,w1); shading interp; colorbar
title('Flexure')
xlabel('Longitude (km)')
ylabel('Latitude (km)')

stress_depth=8000;
Yf=(-.5*Te)+stress_depth; % this sets the depth below the surface
dw2=del2(w1,8.4e3,3.6e3);
Sigma=(1e-6*E*Yf/(1-v^2)).*dw2;

figure(5); surf(gx,gy,Sigma); shading interp; colorbar
title(['Flexural Stress (MPa) at depth (km) = ',num2str(stress_depth/1000)])

figure(6);
[px,py] = gradient(Sigma,8.4e3,3.6e3);
pcolor(gx,gy,Sigma); shading interp; colorbar;
hold on;
v=[0];
contour(gx,gy,ZI,[0 0],'k');
quiver (tx,ty,px,py); hold off;
title('Flexural Stress (MPa) at top of plate')
xlabel('Longitude (km)')
ylabel ('Latitude (km)')
Appendix B: Matlab Script for Figure 9

% Govers Med Miocene Reconstruction

clear all;

load Mediter3.txt;
long=Mediter3(:,1);
lat=Mediter3(:,2);
ele=Mediter3(:,3);

y=lat*111;
x=long.*(2*pi*6371*cosd(lat))/360;

%
c=1;
a=1;
for c=1:length(y)
    if (((y(c))>3000) && ((y(c))<5500)&& ((x(c))<3500) && ((x(c))>-500))
        y(a)=y(c);
        x(a)=x(c);
        elev(a)=ele(c);
        a=a+1;
    end
end

%

tx=(min(x)):1:(max(x));
ty = (min(y)):1:(max(y));
gx=linspace(min(x),max(x),256);
gy=linspace(min(y),max(y),256);

[XI,YI]=meshgrid(gx,gy);
ZI=griddata(x,y,elev,XI,YI);

figure(2);
pcolor(gx,gy,ZI); shading interp; colorbar;
hold on;
v=[0];
contour(gx,gy,ZI,[v v],'k'); hold off;
xlabel('Longitude(km)')
ylabel('Latitude(km)')
title ('Current Topography of Study Region(m)')

%
% just water
ZI2=ZI;
nn=isnan(ZI2);
l=ZI2>0;
b=ZI2<-125;
ZI2(l)=0;
ZI2(b)=-125;
ZI2(nnn)=0;

% flexure
%======================================
Te=18e3; % km elastic thickness

g=9.81; % gravity
v=0.25; %Poisson's ratio
E=56e10; %Young's modulus

npts=256;
D=E*Te^3/(12*(1-v^2)); % flexural parameter

thick=30e3; % m crust thickness

rho_load = 2700;
rho_infill = 2500;
rho_water = 1030;
rho_mantle = 3300;

hk_sea = fft2(ZI2);

ty = 2*pi/3.6e3*(0:1:128)/256;
tx = 2*pi/8.4e3*(0:1:128)/256;

% equations from watt's book
[kx,ky]=meshgrid(tx,ty);

kk = sqrt(kx.^2+ky.^2);
D=E*Te^3/(12*(1-v^2)); % flexural parameter

wk_sea = (rho_water).*(g./(kk.^4.*D+(rho_mantle)*g)).*hk_sea;
w1 = real(ifft2(wk_sea));

depv=1;
tdarray= ones(256,256);
tdarray2= ones (256,256,256) ;

for depv = 1:256

    stress_depth=(25600)-(depv*100);

    Yf=(-.5*Te)+stress_depth;
dw2=del2(wl,8.4e3,3.6e3);
tdarray=(1e-6*E*Yf/(1-v^2)).*dw2*(2/10);
tdarray2(:,:,depv)=tdarray;
end

figure(1);
slice(tdarray2, 125, 125, 236); shading interp; colorbar;
title ('Flexural Stress (MPa)')
zlabel ('Depth (m)')