An Application of Theory: Analysis of Low Frequency Passive Radiator and Ported Systems in a Spherical Loudspeaker Enclosure

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Abstract

The main goal of this study was to fabricate a cost-effective low frequency physical representation (i.e. a loudspeaker) of the theoretical formulation for a spherical cap set in a rigid sphere and subsequently compare the acoustic characteristics (i.e. directivity, sound pressure level, etc.) governed by the theory with experimental measurements. Computational multi-domain modeling was utilized to predict loudspeaker system performance where the theoretical formulation was insufficient (i.e. electrical impedance). A combined analysis utilizing the theoretical formulation and multi-domain modeling offers a broader understanding for assessing loudspeaker system performance. Various methods for improving low-frequency performance of a base loudspeaker design are implemented via a modular design concept and are further assessed in this study by utilizing multi-domain modeling via the Simscape™ (MathWorks®) platform.

0 Introduction

A large percentage of the loudspeakers commercially available for home-theater and studio applications are rectangular in shape, mainly due to the ease of design and manufacturability; however, rectangular loudspeaker enclosure designs suffer from edge diffraction effects which “color” the radiated sound and are therefore deemed less than optimal. Harry Olson was notably the first to study the effect of diffraction due to enclosure shape in his landmark 1951 JAES article entitled “Direct Radiator Loudspeaker Enclosures” [1], wherein he determined the effect of diffraction for the on-axis frequency response of an active radiator mounted in 12 different loudspeaker geometries, of which included a sphere, a cube, a cylinder, and a rectangle, among others. Intuition allows for the postulation that the sphere would result in the flattest frequency response due to the absence of edge diffraction effects. Harry Olson determined that the sphere indeed had the flattest frequency response with an average deviation in sound pressure level of ±0.5 dB above the baffle step response transition frequency, whereas other loudspeaker geometries exhibited a varying “rippling” in their frequency response due the presence of edge diffraction. One can conclude that a spherical loudspeaker, when implemented correctly, would result in an optimal frequency response; therefore, the goal of this study was to fabricate a cost-effective low-frequency spherical loudspeaker whose performance could be predicted by both a theoretical formulation and the utilization of computational multi-domain modeling. The theoretical formulation for a spherical cap set in a rigid sphere has been well documented for many decades and has been studied in extensive detail in various capacities and applications and therefore serves as a basis for the prediction of performance for a physical representation of an active radiator in a spherical enclosure. Much of the pertinent information for loudspeaker system performance can be gained from electrical impedance measurements which are not considered in the ideal theoretical formulation and therefore computational multi-domain “lumped-element” modeling can be implemented to further predict the performance of a loudspeaker system.

Firstly, the theoretical formulation for a spherical cap set in a rigid sphere is expounded upon where Theoretical Acoustics (1968) [2] by P. Morse and K. Ingard is used as a trusted source to predict the purely acoustical characteristics of a spherical loudspeaker. Secondly, the design and fabrication process for the practical implementation of an active radiator (and other loudspeaker configurations) in a spherical enclosure is discussed. Thirdly, the implementation of lumped-element multi-domain models through the use of Simscape™ (MathWorks®) simulation software to simulate the electrical impedance and sound pressure level response curves is discussed. Both the theoretical formulation and the lumped-element models offer a combined, more detailed picture of the loudspeaker performance, as each is adept at predicting certain aspects of this performance. For example, a capacitance lumped-element can be used to represent an enclosure volume, but the dimensionality of the enclosure cannot be readily adopted within a lumped-element model. Fourthly, laboratory measurements of electrical impedance and the subsequent performance results derived from said measurements, as well as anechoic chamber measurements of directivity and sound pressure level, will be discussed. Lastly, predictions from the theoretical formulation (i.e. directivity, sound pressure level, etc.) and Simscape™ multi-domain models (i.e. electrical impedance) will be compared against laboratory measurements.
1 Background

1.1 Background

The main design goal of this study was to fabricate a cost-effective low frequency physical representation of the theoretical formulation for a spherical cap set in a rigid sphere, a formulation that has been well documented for many decades and has been studied in extensive detail in various capacities and applications. A complete theoretical formulation for a plane piston set in a sphere can be found in The Foundations of Acoustics (1971) [3] by E. Skudrzyk; however, this theoretical formulation is quite complex and beyond the scope of this study. This complexity arises due to the absence of a coordinate system that fits the boundary conditions for the geometry of a plane piston set in a sphere, resulting in a solution that cannot be represented by spherical Bessel functions and their associated orthogonality characteristics. Fortunately, there exists theoretical formulations for a spherical cap set in a rigid sphere for which the characteristic results (i.e. directivity, sound pressure level, etc.) agree considerably well at low frequencies when compared to that of a plane piston set in a sphere. One such theoretical formulation, which will be used as the basis of this study, is presented in the classic textbook Theoretical Acoustics (1968) by P. Morse and K. Ingard for “radiation from a piston set in a sphere”. This theoretical formulation may be extended to include piston geometries other than that of a spherical section or “cap”, such as a typical loudspeaker diaphragm that employs an inverted cone shape to increase its mechanical stiffness. P. Morse and K. Ingard regard,

“The radiation from such a cone would naturally differ from that from a section of a sphere; but it turns out that the average radiation impedance on a piston is approximately the same, no matter what its shape, as long as the circumference is not changed and as long as the volume of the mounting case is not changed.” [2]

Therefore, the fabrication of an approximately spherical “baffle” for the housing of a typical loudspeaker diaphragm may be a suitable physical representation of the theoretical formulation proposed by P. Morse and K. Ingard. Fig. 1 represents a transformation from the geometry of the theoretical formulation to a practical physical representation of a loudspeaker housed in a spherical “baffle” of the same circumference and volume.

1.2 Theoretical Formulation

The theoretical formulation presented by P. Morse and K. Ingard in Theoretical Acoustics (1968) for the “radiation from a piston set in a sphere” assumes a spherical section or “cap” of radius $a \sin(\beta_0/2)$ and surface area $4\pi a^2 \sin^2(\beta_0/2)$ set in a rigid sphere of radius $a$ centered at the origin, where $\beta_0$ is the angle (in degrees) between the z-axis for any line connecting the origin to a point on the “rim” of the spherical “cap” (see Fig. 2). If $\beta_0$ is not sufficiently large, the velocity amplitude distribution as a function of the angle $\beta$ on the surface of the rigid sphere is approximately:

$$U(\beta) = \begin{cases} u_0 & 0 \leq \beta < \beta_0 \\ 0 & \beta_0 < \beta \leq \pi \end{cases}$$

where $u_0$ is the piston velocity (m/s) normal to the surface of the spherical “cap” and $\beta$ is the elevation angle (see Fig. 2).

This axially symmetric velocity profile allows for a simplified form of the general pressure expression:

$$p(r,\beta,\theta) = -ipc \frac{u_0}{2} \sum_{n=0}^{\infty} \left[ P_{n-1}(\cos(\beta_0)) - P_{n+1}(\cos(\beta_0)) \right] \frac{P_n(\cos(\theta))}{h_n^{(2)}(k\alpha)} [\text{Pa}]$$

where $\rho$ is the density of air (kg/m$^3$), $c$ is the speed of sound in air (m/s), $k = \omega/c$ (m$^{-1}$), $\omega$ is the radial frequency of the applied excitation signal (rad/s), $r$ is the radius of evaluation (m) (see Fig. 2), $P_n$ is the Legendre polynomial of order $n$ ($n = 0, 1, 2, ...$), $\theta$ is the azimuthal angle (in degrees) (see Fig. 2), and $h_n^{(2)}$ and $h_n^{(2)'}$ are the spherical Hankel function and its derivative of order $n$ ($n = 0, 1, 2, ...$). From this general pressure expression, one can obtain expressions for directivity and sound pressure level:

$$D(r,\theta) = \sum_{n=0}^{N} i^n [P_{n-1}(\cos(\beta_0)) - P_{n+1}(\cos(\beta_0))] \frac{P_n(\cos(\theta))}{h_n^{(2)}(k\alpha)}$$

Figure 2 Geometry and notation for a spherical cap set in a rigid sphere.

$\text{Figure 1}$ 3-dimensional CAD (Autodesk® Fusion 360) representations of a spherical section or “cap” (left) and a typical loudspeaker diaphragm profile (right) set in their respective spherical “baffles”.
where $N \approx 2ka$, $L_p$ is the sound pressure level (dB re 20 μPa), and $p_{ref}$ is the reference pressure at sea level (20 μPa). Fig. 3 depicts the directivity plots for six third-octave bands for values of $\beta_0 = 18.25^\circ$ and $\alpha = 0.1397$ m (11 in.). These values represent the dimensions of a simplified geometry of the base loudspeaker configuration analyzed in this study which will be discussed in greater detail in §2.2. Inspection of directivity plots comparing a plane piston set in a rigid sphere and a spherical cap set in a rigid sphere in The Foundations of Acoustics (1971) by E. Skudrzyk give a clear indication that the simplified theoretical formulation presented by P. Morse and K. Ingard adopted in this study is sufficient for subsequently obtaining predicted experimental measurements.

The directivity plot in Fig. 3 shows that there is fairly uniform directivity up to 500 Hz, above which there is a tendency for the pressure to become increasingly focused in a narrow region normal to the piston face. The directivities of the physical representation will be compared to the theoretical directivities in the §4.2. It can be seen that the sound pressure level at the six third-octave band frequencies of interest exhibit the same trends as those presented for the directivities presented in Fig. 3. It should be noted that general pressure expression requires a uniform value of $u_0$ and $r$ to subsequently predict the exact sound pressure levels to be expected at a given third-octave band frequency and distance. For the graph in Fig. 4 a conservative value of $1 \times 10^4$ m/s for $u_0$ was utilized, with an approximate average value attainable via an accelerometer measurement or being inferred by measurement of the pressure in an anechoic environment, the latter of which will be utilized. The graph in Fig. 4 serves as a visual aid and does not infer the exact decibel levels specified for this reason.

$\begin{align*}
L_p &= 20 \log_{10} \left( \frac{p(r, \beta, \theta)}{p_{ref}} \right) \text{ [dB, re 20 μPa]} \\

\end{align*}$

**Figure 3** Normalized directivity plots for third-octave band frequencies of interest.

**Figure 4** Sound pressure level vs. angle for six third-octave band frequencies of interest for uniform velocity ($u_0$) of $1 \times 10^4$ m/s.

Of practical application to the loudspeaker designer is acoustic intensity and power output over the frequency range of operation. P. Morse and K. Ingard further provide equations and their intermediaries for intensity and power, which are provided for convenience. Theoretical Acoustics (1968) by P. Morse and K. Ingard may be consulted for further instruction on the derivation of these equations.

Power:

$$\Pi = \frac{\rho c^4}{4\pi f_2} \sum_{m=0}^{N} \frac{u^2_i}{(2m+1)\beta_m} \text{ [W]}$$

Intensity:

$$I_r \approx \rho c u_0^2 \frac{a^2}{r^2} F_r(\beta) \text{ [W/m}^2\text{]}$$

Angle-distribution function:

$$F_r(\beta) = |\psi(\beta)|^2$$

where $\psi(\beta) = \frac{1}{\kappa a} \sum_{m=0}^{N} \frac{u_m}{u_0 \beta_m} P_m(\cos(\beta)) e^{-i\delta_m - \frac{1}{2}i\pi(m+1)}$

Phase-angle coefficients:

$$\delta_m = \tan^{-1} \left( \frac{(m+1)\beta_m - m\beta_{m-1}}{(m+1)\beta_m + m\beta_{m-1}} \right)$$

Amplitude coefficients:

$$B_m = \frac{j m \beta_m - (m+1) \beta_{m+1}}{(2m+1) \cos(\beta_m)}$$

Velocity coefficients:

$$U_m = \frac{1}{2} u_0 \left[ P_{m-1}(\cos(\beta_0)) - P_{m+1}(\cos(\beta_0)) \right]$$

where $j_m$ is the spherical Bessel function of order $m (m = 0, 1, 2, ...)$ and $n_m$ is the spherical Neumann function of order $m (m = 0, 1, 2, ...).$ As there is no such readily available graphical interpretations of the phase-angle and amplitude coefficients for a spherical cap set in a rigid sphere in the cited literature, they are provided in Fig. 5 and Fig. 6 for visual inspection so as to provide an intuitive basis for their effect on the intensity and power for a given frequency of interest. Tabular values of the first 10 coefficients for the phase-angle and amplitude coefficients are available in the Appendix. It can be seen from visual inspection that for small values of $ka$, the first few amplitude coefficient terms ($B_m$) for the
expansions needed to generate power curves can be neglected due to a rapid increase in value as \( m \) increases.

![Figure 5](image_url)  
**Figure 5** Phase-angle coefficients for the radiation from a general spherical source. *Note.* The dotted black lines denote the ka values corresponding to the six third-octave band frequencies of interest.

![Figure 6](image_url)  
**Figure 6** Amplitude coefficients for the radiation from a general spherical source. *Note.* The dotted black lines denote the ka values corresponding to the six third-octave band frequencies of interest.

## 2 Design

### 2.1 Loudspeaker Design & Fabrication

In order to simplify analysis, a low frequency system was adopted through the use of a “micro” subwoofer (Tang Band® W3-2088S0F). This was a conscious design choice so that measured results (i.e. directivity, sound pressure level, etc.) would be comparable to the theoretical formulation, and minimize the measured effects of diffraction due to the active radiator diaphragm not being perfectly flush with the outside edge of the constructed spherical “baffle” (see Fig. 1). A preliminary conceptual design phase for the practical implementation of a low frequency radiator set in a rigid spherical “baffle” resulted in the selection of 2 hollow hemispherical bodies (IKEA® Blanda Matt bowls) whose material and geometrical properties allowed for the satisfaction of both the rigid boundary condition imposed by the theoretical formulation, and the housing of the selected low frequency radiator. The utilization of 3D printing services via Pennsylvania State University’s MakerBot™ Innovation Center for the custom fabrication of a spherical baffle was considered during the preliminary conceptual design phase; however, due to dimensional, structural, and monetary constraints, this design route was discarded. Although the IKEA® Blanda Matt bowls do not completely form an exact spherical shape when combined edge-to-edge to create a spherical “baffle”, they can be utilized as a cost-effective solution to obtain the desired design goal. The construction of the loudspeaker was a relatively straightforward process with most of the materials being available from online distributors. Additional custom parts were fabricated through the MakerBot™ Innovation Center, which included various radiator “bushings” and a cylindrical port. Woodworking services were also provided by Pennsylvania State University’s Engineering Shop Services and assembly was conducted in the Acoustics program research facilities (Research West Building). Table 1 lists a complete bill of materials and associated costs used for the construction of various loudspeaker configurations that were used in the present study. Without considering the cost of labor and services rendered, the total cost for all loudspeaker configurations was approximately $150; therefore, satisfying the cost-effective design goal. It should be noted that custom 3D printed materials such as the cylindrical port were printed for free through the MakerBot™ Innovation Center; therefore, non-access to these services would result in a slightly greater overall project cost.

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Part Name</th>
<th>Quantity</th>
<th>Price/Unit</th>
</tr>
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<tr>
<td>A</td>
<td>IKEA® Blanda Matt Bowl</td>
<td>2</td>
<td>$19.99</td>
</tr>
<tr>
<td>B</td>
<td>Tang Band® W3-2088S0F</td>
<td>1</td>
<td>$45.70</td>
</tr>
<tr>
<td>[C]</td>
<td>Tympahany® Peerless 830878 Cap</td>
<td>1</td>
<td>$10.21</td>
</tr>
<tr>
<td></td>
<td>Cylindrical Port</td>
<td>1</td>
<td>n/a</td>
</tr>
<tr>
<td>D</td>
<td>SpeakON Connector</td>
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<td>$2.59</td>
</tr>
<tr>
<td>E</td>
<td>Wing Turn Latch</td>
<td>4</td>
<td>$4.72</td>
</tr>
<tr>
<td>F</td>
<td>D-Profile Weatherstrip</td>
<td>17 ft.</td>
<td>$7.34</td>
</tr>
<tr>
<td>G</td>
<td>Acousta-Stuf Polyfill</td>
<td>1 lb.</td>
<td>$11.50</td>
</tr>
<tr>
<td>H</td>
<td>#6 ½” Pan-Head Phillips Screws</td>
<td>16</td>
<td>$1.18/13-Pack</td>
</tr>
<tr>
<td>I</td>
<td>#8 ½” Pan-Head Phillips Screws</td>
<td>4</td>
<td>$1.18/13-Pack</td>
</tr>
<tr>
<td>J</td>
<td>#6 1-½” Pan-Head Phillips Screws</td>
<td>4</td>
<td>$1.18/7-Pack</td>
</tr>
<tr>
<td>K</td>
<td>#6 ½” Pan-Head Phillips Screws</td>
<td>4</td>
<td>$1.18/4-Pack</td>
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<tr>
<td>L</td>
<td>PR Bushing</td>
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<tr>
<td>M</td>
<td>AR Bushing</td>
<td>1</td>
<td>n/a</td>
</tr>
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</table>

*Note.* A bill of materials and associated costs for various loudspeaker configurations utilized in this study. The [C] denotes parts that are modular and exchangeable. The identifier denotes the location of a part in the loudspeaker assemblies shown in Fig. 7, 8, 9. *Note.* AR – Active Radiator, PR – Passive Radiator.
2.2 Loudspeaker Configurations

As a potent curiosity, the implementation of a passive radiator versus a port to increase radiation efficiency, low frequency response, and “backside” directivity (i.e. opposite the active radiator location) of the base loudspeaker configuration is assessed in this study via a modular design concept, whereby the parts labeled with the identifier [C] in Table 1 (and concurrently Fig. 7, 8, 9) are readily interchangeable. It has long been known that the inclusion of either a passive radiator or a port, depending on the system they are implemented in, can result in the aforementioned loudspeaker performance attributes; however, both have their advantages and disadvantages. A few of these advantages and disadvantages are listed below. For a more detailed explanation of both design considerations one can consult The Loudspeaker Design Cookbook (2006) [4] by Vance Dickason.

Port Advantages/Disadvantages [4] [5]:
- Reduced active radiator diaphragm excursion near the enclosure resonance frequency, resulting in better power handling and reduced modulation distortion
- Sensitive to misaligned parameters (i.e. enclosure geometry, port location, etc.)
- Increased transient stability due to lower and smoother cutoff frequency compared to passive radiator designs
- Increase of 3 dB in overall radiation efficiency compared to a closed-box design

Passive Radiator Advantages/Disadvantages [4] [5]:
- Elimination of coloration artifacts from ported designs (i.e. pipe resonances, air flow noise, high frequency reflections, etc.)
- Practical application for relatively small enclosures that require port lengths in excess of internal enclosure dimensions
- Reduced transient stability due to a higher and steeper cutoff frequency compared to ported designs
- Greater overall system losses compared to ported designs

The three loudspeaker configurations analyzed in this study were:

1) Active (Tang Band® W3-2088S0F) radiator in a spherical baffle (see Fig. 7)

2) Active (Tang Band® W3-2088S0F) and passive (Tymphany® Peerless 830878) radiator(s) in a spherical baffle (see Fig. 8)

3) Active (Tang Band® W3-2088S0F) radiator and a cylindrical port in a spherical baffle (see Fig. 9)
2.3 Low Frequency Active and Passive Selection

As was stated in the previous section, a conscious decision was made to utilize a low frequency system through the employment of a “micro” subwoofer acting as the active radiator. The Tang Band® W3-2088S0F was selected for its compact form factor, solicited “ultra-linear” excursion properties, price, and relatively low resonance frequency. Table 2 lists pertinent Tang Band® W3-2088S0F specification parameters obtained from the company specification sheet with additional Thiele/Small parameters included. Equations for the calculation of Thiele/Small parameters have been well documented in a variety of journal articles and scholarly texts. Testing Loudspeakers (1998) [5] by J. D’Appolito was used as a reference for these equations. The Tymphany® Peerless 830878 was selected as the passive radiator mainly for its dimensional characteristics as it is rated as having the same nominal diaphragm size as the Tang Band® W3-2088S0F.

Table 2: Tang Band® W3-2088S0F Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
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<td>BL</td>
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</tr>
<tr>
<td>C_MS</td>
<td>7.66×10⁻⁴</td>
<td>kg</td>
</tr>
<tr>
<td>fS</td>
<td>45.0</td>
<td>Hz</td>
</tr>
<tr>
<td>L_VC</td>
<td>1.8×10⁻⁴</td>
<td>H</td>
</tr>
<tr>
<td>M_MS</td>
<td>1.796×10⁻²</td>
<td>kg</td>
</tr>
<tr>
<td>Q_ES</td>
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</tr>
<tr>
<td>Q_MS</td>
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<tr>
<td>Q_TS</td>
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<tr>
<td>R_E</td>
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<td>ohm</td>
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<td>R_ES</td>
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<td>ohm</td>
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<td>R_MS</td>
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<td>N-s/m</td>
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<td>S_D</td>
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<td>S0</td>
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<td>dB SPL/2.83 V/1 m</td>
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<td>S_P</td>
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<td>dB SPL/2.83 V/1 m</td>
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<tr>
<td>V_AS</td>
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<td>η₀</td>
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<td>%</td>
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Note. Tang Band® W3-2088S0F specification parameters obtained from the company specification sheet with additional Thiele/Small parameters included.

Table 3: Tymphany® Peerless 830878 Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
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</thead>
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<tr>
<td>C_MS</td>
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</tr>
<tr>
<td>fS</td>
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<td>Hz</td>
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<tr>
<td>M_MS</td>
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</tr>
<tr>
<td>S_D</td>
<td>5.02×10⁻³</td>
<td>m²</td>
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</table>

Note. Tymphany Peerless 830878 specification parameters obtained from the company specification sheet.

2.4 Port Fabrication

It is well known that the implementation of a vented-box system is quite sensitive due to “misaligned” parameters, as minute incorrect sizing of the port geometry can lead to unwanted decreases or increases in bass response. “Alignment” in this context is the effective implementation of an enclosure size and port tuning combination that produces a desired flat frequency response, of which there are 15+ well-defined alignment categories for such an effective vented-box system. For the third loudspeaker configuration, the port was custom fabricated through the MakerBot® Innovation Center, whereby the dimensions of the port were maximally selected given the dimensional constraints of the spherical enclosure, whereby none such “alignments” were considered.

![Figure 10 A 2-dimensional CAD (Autodesk® Fusion 360) representation of the custom 3D printed port.](image)

The utilization of a port to increase bass performance was a potent curiosity design afterthought, for which the performance would be assessed via experimental measurement. The constraints on the size of the port allowed for a port radius of 1.375 in. (3.5×10⁻² m) and a port length of 5.32 in. (0.1351 m); therefore, the total acoustic mass of the port, including end corrections and considering one end of the port to be flanged is given by the equation:

\[ M_{AT} = \frac{\rho}{A_p} (l + 0.84\sqrt{A_p}) \left[ \frac{kg}{m^4} \right] \]

where \( \rho \) is the density of air (kg/m³), \( A_p \) is the cross-sectional area of the port (m²), \( l \) is the length of the port (m), and 0.84 is a correction factor for a port with a flanged opening on one side; thereby resulting in a total acoustic mass of 59.1 kg/m³ and an equivalent mechanical mass of 8.67×10⁻⁴ kg. Subsequently, this mechanical mass can be used to predict the shift down in resonance frequency.

3 Multi-domain Modeling

3.1 Simscape™

Simscape™ simulation software enables the user to model multi-domain physical systems within the Simulink® (MathWorks®) environment. With Simscape™, the user can build physical component models based on physical connections that directly integrate with block diagrams and other modeling paradigms. The creation of an accurate multi-domain model of a physical system offers valuable insight into the interaction between different physical systems (i.e. electrical, mechanical, acoustical, etc.) as well as offering a means of predicting system performance and subsequent system optimization. Simscape™ was used to develop multi-
domain lumped-element models for each the three loudspeaker configurations analyzed in this study.

### 3.2 Lumped-Element Models

Mathematically speaking, a lumped-element model is a simplification of a “state-space” model of a system into a system of finite dimension, whereby the partial differential equations (PDEs) that govern the continuous time and space model of a physical system are “linearized” into a system of finite ordinary differential equations (ODEs). For example, the lumped-element model of an electronic circuit is comprised of idealized embodiments of inherent electronic circuit properties (i.e. resistances, capacitances, inductances, etc.) joined by a network of perfectly conducting wires, wherein a lumped-element is valid when the circuit’s “characteristic” length is much smaller than the circuit’s “operating” wavelength. The same lumped-element concept may be further extended to encompass other physical systems such as in the mechanical and acoustic domains as long as the lumped-element approximation is valid in a respective domain.

The simplifying assumptions for a lumped-element model in the mechanical domain is that all “bodies” are rigid and that all interactions between these rigid bodies are mediated by joints, springs, and dampers. In the acoustic domain, a lumped-element model consists of acoustical properties that may be attributed to electronic circuit components, for which an acoustic element must have dimensions smaller than the acoustic wavelength of interest. For example, a “reflex” port may be attributed to electronic circuit components, for which an acoustic element must have dimensions smaller than the acoustic wavelength of interest. For example, a “reflex” port filled with air having an acoustic mass may be approximated as an inductor in an electronic circuit, and a closed loudspeaker enclosure having a volume may be approximated as a capacitor whose value is proportional to that volume. Since a typical dynamic loudspeaker is a multi-domain system (i.e. electrical, mechanical, and acoustical), the utilization of a lumped-element model to represent the interaction between all domains involved can provide valuable insight into the complex interaction between domains and the overall performance of the system, as well as allowing the user to alter specific parameters to optimize system performance.

### 3.3 Loudspeaker Configuration Models

For this study, all loudspeaker configurations in Fig. 7, 8, 9 were modeled using the Simscape™ simulation software, wherein the electrical, mechanical, and acoustical domains were constructed from lumped elements whose values were either obtained from measurement, product specifications sheets, or inferred by linear analysis of the multi-domain model. Table 4 is a complete list of all lumped-element components sorted by domain (i.e. electrical, mechanical, acoustical, etc.) for all the loudspeaker configurations analyzed in this study. In sections §3.3.1, 2, 3, each loudspeaker configuration’s lumped-element model will be expounded upon systematically by domain and component type, with full Simscape™ block diagrams of each loudspeaker configuration available in the Appendix for detailed reference.

### 3.3.1 Active Radiator in a Spherical Baffle

The base loudspeaker configuration modeled in this study was that of an active radiator (Tang Band® W3-208850F) housed in a spherical baffle, a configuration that is a representation of the theoretical formulation for a spherical cap set in a rigid sphere. All lumped-element components that define the modeling of this loudspeaker configuration are denoted by AR in Table 4. Starting in the electrical domain, an active radiator’s electrical properties are mainly governed by two components, that of “voice-coil inductance” \(L_{VC}\) and “DC resistance” \(R_{DC}\). The “voice-coil inductance” \(L_{VC}\) of an active radiator is measured in millihenries (mH) and the industry standard is to measure its nominal value for a sinusoidal input signal of 1000 Hz. The effect of “voice-coil inductance” for a typical active radiator can be seen on the active radiator’s electrical impedance vs. frequency curve, where above approximately 1000 Hz, the impedance curve’s slope gradually increases. The “DC resistance” \(R_{DC}\) or “DCR” represents the DC resistance of the coil of wire used for the voice coil and is typically slightly below its nominally specified value. Fig. 11 depicts the electrical domain for an active radiator, wherein an “impedance analogy” is utilized for which voltage is the “potential” variable and current is the “flow” variable. The transformation from the electrical domain to the mechanical domain is achieved through the generation of a magnetic field that is proportional to the BL product (“force factor”), a quantity that is a vector cross product of the MMF (magneto motive force) and the length of coil used in the voice coil construction.

In the mechanical domain, an active radiator’s mechanical properties may be approximated by a simple harmonic oscillator, wherein the electrical equivalent circuit of the mechanical domain may be represented as a circuit containing an inductance, resistance, and capacitance representing the effective mechanical mass \((M_{MS})\), suspension resistance \((R_{MS})\), and suspension compliance \((C_{MS})\) of the active radiator assembly. The transformation from the mechanical domain to the acoustical domain is achieved through the mechanical motion of the active radiator’s surface area coupled to surrounding medium, in this case, air; therefore, the transformation “coefficient” is the effective surface area \((S_{0})\) of the active radiator. In the acoustical domain, the active radiator may be represented by an acoustic radiation impedance that is specific to the boundary conditions of the active radiator.
For a piston set in an infinite baffle, the radiation impedance of an active radiator may be approximated by the equivalent electrical circuit in Fig. 13, whose lumped-component values are computed from equations that approximate an exact theoretical formulation for a piston set in an infinite baffle. The circuit and associated equations were obtained from Acoustics: Sound Fields and Transducers (2012) [6] by L. Beranek and T. Mellow and are presented here for convenience:

\[
R_{A1} = 0.1404 \frac{\rho c}{a^2} \left[ \frac{N s}{m^2} \right]
\]

(12)

\[
R_{A2} = 0.318 \frac{\rho c}{a^2} \left[ \frac{N s}{m^2} \right]
\]

(13)

\[
C_{A1} = 5.94 \frac{a^2}{\rho c^2} \left[ \frac{m^5}{N} \right]
\]

(14)

\[
M_{A1} = \frac{8}{3} \frac{\rho}{\pi a^2} \left[ \frac{k g}{m^4} \right]
\]

(15)

where \( a \) is the effective radius of the active radiator (m), \( \rho \) is the density of air (kg/m\(^3\)), and \( c \) is the speed of sound in air (m/s). The spherical baffle in the acoustical domain may be represented by an acoustic compliance (\( C_{A1} \)), whose value is proportional to the volume of the inside spherical enclosure. The acoustic compliance of an enclosure is obtained by the following equation:

\[
C_{A1} = \frac{V}{\gamma p_0} \frac{m^5}{N}
\]

(16)

where \( V \) is the volume of the enclosure (m\(^3\)), \( \gamma \) is the ratio of specific heats for air, and \( p_0 \) is standard atmospheric pressure (Pa). The complete Simscape™ multi-domain block diagram for the Active Radiator in a Spherical Baffle can be referenced in the Appendix (Fig. A.1).

### 3.3.2 Active Radiator and Passive Radiator in a Spherical Baffle

The second loudspeaker configuration modeled in this study employed a passive radiator (see Fig. 8) as being the first of two means to increase the low-frequency performance of the base loudspeaker configuration. The multi-domain model for this loudspeaker configuration builds on the model for the Active Radiator in a Spherical Baffle, whereby the passive radiator is represented by a simple harmonic oscillator (see Fig. 12) in the acoustical domain and a radiation impedance (see Fig. 13) in the acoustical domain, coupled by the back volume of the spherical enclosure. The complete Simscape™ multi-domain block diagram for the Active Radiator and Passive Radiator in a Spherical Baffle can be referenced in the Appendix (Fig. A.2).

### 3.3.3 Active Radiator and a Port in a Spherical Baffle

The third loudspeaker configuration modeled in this study employed a cylindrical port (see Fig. 10) as being the second of two means to increase the low-frequency performance of the base loudspeaker configuration. The multi-domain model for this loudspeaker configuration builds on the model for the Active Radiator in a Spherical Baffle, whereby a port is represented in the acoustical domain as a lossy 2-port network (see Fig. 14) and a radiation impedance (see Fig. 13). The equivalent circuit for a lossy 2-port network is similar to a transmission line model whose components represent the radiation losses associated with thermal (subscript \( T \)) and viscous (subscript \( V \)) losses along the length of the port walls. The circuit and associated equations were obtained from Acoustics: Sound Fields and Transducers (2012) by L. Beranek and T. Mellow and are presented here for convenience:

\[
R_p = \frac{8 \mu l}{(1+4\beta a )a^2} \left[ \frac{k g}{s m^2} \right]
\]

(17)

\[
M_p = \frac{1+33 \beta u}{1+4\beta u} \frac{4}{3} \frac{I_k}{\pi p_0^2} \left[ \frac{kg}{m^4} \right]
\]

(18)

\[
R_T = \frac{(1+33 \beta u) \gamma P_0 a^2}{6(\gamma-1)\kappa p_0^2} \left[ \frac{kg}{s} \right]
\]

(19)

\[
C_T = (\gamma - 1) C_0 \left[ \frac{kg}{s^2} \right]
\]

(20)

\[
C_0 = \frac{1}{\gamma p_0} \left[ \frac{s^2}{kg} \right]
\]

(21)

where, \( \mu \) is the viscosity coefficient of air (N-s/m\(^2\)), \( l \) is the length of the port (m), \( a \) is the radius of the port (m), \( \rho \) is the density of air (kg/m\(^3\)), \( \gamma \) is the ratio of specific heats for air, \( p_0 \) is standard atmospheric pressure (Pa), \( \kappa \) is the thermal conductivity of air (N-s/K), \( T_0 \) is standard atmospheric temperature (K), and \( B_p \) and \( B_e \) are coefficients defined by:

\[
B_p = \left( \frac{2}{\beta_T} \right) \left( \frac{\frac{1}{\alpha_T} - 1}{\alpha_T} \right) K_n
\]

(22)

\[
B_e = \left( \frac{\frac{1}{\beta_e} - 1}{\alpha_e} \right) K_n
\]

(23)

where \( K_n \) is the Knudsen number, \( \beta_T \) is the Prandtl number, and \( \alpha_T \) and \( \alpha_e \) are “accommodation” coefficients with a specified value of 0.9. It should be noted that each of the components for the lossy 2-port network are to be transformed from the mechanical domain units in which they are presented in...
3.3.5 Spherical Enclosure

The “Baffle Step Filter” component (see Table 4) in the electrical domain represents the baffle step response of the spherical enclosure. All loudspeaker enclosure’s exhibit a baffle step response or “diffraction loss” depending on the geometry of the enclosure, in which there is an average increase of 6 dB from low to high frequencies. Low frequencies whose large wavelengths compared to enclosure dimensions essentially radiate into a “full space”, whereas high frequencies whose small wavelengths compared to enclosure dimensions essentially radiate in a “half-space”; therefore, there is a transition from a “full-space” to a “half space” known as the baffle step response. H. Olson’s famous work, entitled “Direct Radiator Loudspeaker Enclosures” explores how different loudspeaker enclosure geometries (i.e. sphere, hemisphere, cylinder, etc.) affect the far-field sound pressure level due to their respective baffle step responses. It was concluded that a spherical enclosure offers the smoothest baffle step response as it does not exhibit abrupt boundary conditions that result in “diffraction loss”. The baffle step response of a spherical enclosure may be modeled as the equivalent circuit in Fig. 15, in which \( R_{BS1} \) and \( R_{BS2} \) are the same value and \( C_{BS} \) is governed by the following equation:

\[
C_{BS} = \frac{\sqrt{2} \cdot \pi R_{BS} f_{3\,dB}}{2 f_{3\,dB} \pi R_{BS} f_{3\,dB}} \quad [F]
\]  

(24)

where \( R_{BS} \) (ohm) is the summation of \( R_{BS1} \) (ohm) and \( R_{BS2} \) (ohm) and \( f_{3\,dB} \) (Hz) is inversely proportional to the spherical baffle diameter whose value is calculated by:

\[
f_{3\,dB} = \frac{115}{W_B} \quad [Hz]
\]  

(25)

where \( W_B \) is the spherical baffle diameter (m) and the value in the numerator represents a conversion factor. Fig. 16 depicts what the baffle step response or “diffraction loss” curve is for a spherical enclosure with a \( W_B \) of 11 in. (0.2794 m) and an \( f_{3\,dB} \) of approximately 400 Hz.

---

**Figure 14** Acoustical domain a lossy 2-port network that includes thermal and viscous loss components, denoted by a subscript T or V. **Note.** Grounding node omitted for space conservation.

**Figure 15** Electrical domain equivalent circuit of the baffle step filter for a spherical enclosure. **Note.** Grounding node omitted for space conservation.

**Figure 16** Baffle step response (“diffraction loss”) for a \( W_B \) of 11 in. (0.2794 m) over the frequency range of interest.

---

**Table 4: Simscape™ Lumped Elements**

<table>
<thead>
<tr>
<th>Domain</th>
<th>Component</th>
<th>Lumped Element</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Baffle Step Filter</td>
<td>( R_{BS1} )</td>
<td>100</td>
<td>ohm</td>
</tr>
<tr>
<td>E</td>
<td>Baffle Step Filter</td>
<td>( R_{BS2} )</td>
<td>100</td>
<td>ohm</td>
</tr>
<tr>
<td>E</td>
<td>AR Blood</td>
<td>( M_{BS} )</td>
<td>2.35x10^2</td>
<td>kg/m^2</td>
</tr>
<tr>
<td>E</td>
<td>AR Material</td>
<td>( k_{MS} )</td>
<td>2532</td>
<td>N/m/m/s</td>
</tr>
<tr>
<td>M</td>
<td>AR Radiation</td>
<td>( R_{MS} )</td>
<td>1.235</td>
<td>N/m/s</td>
</tr>
<tr>
<td>M</td>
<td>PR Radiation</td>
<td>( k_{MS} )</td>
<td>3.85x10^2</td>
<td>kg/m/s</td>
</tr>
<tr>
<td>M</td>
<td>PR Radiation</td>
<td>( k_{MS} )</td>
<td>1.26x10^4</td>
<td>N/m/s</td>
</tr>
<tr>
<td>M</td>
<td>Enclosure with Port</td>
<td>( C_{AB} )</td>
<td>6.63x10^4</td>
<td>m^2/N</td>
</tr>
<tr>
<td>M</td>
<td>Enclosure</td>
<td>( C_{AB} )</td>
<td>6.69x10^4</td>
<td>m^2/N</td>
</tr>
<tr>
<td>A</td>
<td>PR Radiation</td>
<td>( R_{AB} )</td>
<td>4.17x10^4</td>
<td>N/m/s</td>
</tr>
<tr>
<td>A</td>
<td>PR Radiation</td>
<td>( R_{AB} )</td>
<td>9.44x10^4</td>
<td>N/m/s</td>
</tr>
<tr>
<td>A</td>
<td>PR Radiation</td>
<td>( C_{AB} )</td>
<td>2.18x10^9</td>
<td>m^2/N</td>
</tr>
<tr>
<td>A</td>
<td>PR Radiation</td>
<td>( M_{AB} )</td>
<td>8.74</td>
<td>kg/m^4</td>
</tr>
<tr>
<td>A</td>
<td>Port</td>
<td>( R_{AB} )</td>
<td>4.78x10^4</td>
<td>N/m/s</td>
</tr>
<tr>
<td>A</td>
<td>Port</td>
<td>( R_{AB} )</td>
<td>1.08x10^5</td>
<td>N/m/s</td>
</tr>
<tr>
<td>A</td>
<td>Port</td>
<td>( C_{AB} )</td>
<td>1.78x10^9</td>
<td>m^2/N</td>
</tr>
<tr>
<td>A</td>
<td>Port</td>
<td>( M_{AB} )</td>
<td>9.36</td>
<td>kg/m^4</td>
</tr>
<tr>
<td>A</td>
<td>Radiation Impedance</td>
<td>( R_{AT} )</td>
<td>1.04x10^10</td>
<td>N/m/s</td>
</tr>
<tr>
<td>A</td>
<td>Radiation Impedance</td>
<td>( C_{AT} )</td>
<td>3.36x10^10</td>
<td>m^2/N</td>
</tr>
<tr>
<td>A</td>
<td>Radiation Impedance</td>
<td>( M_{AT} )</td>
<td>8.32x10^10</td>
<td>m^2/N</td>
</tr>
</tbody>
</table>

**Note.** A table containing the lumped-element component values for all Simscape multi-domain model configurations, where E – Electrical, M – Mechanical, A – Acoustical, AR – Active Radiator, PR – Passive Radiator, ESA – Effective Surface Area. **Note.** A foreword slash (’/’) between domains denotes a transformation from one domain to another.
3.4 Simscape™ Results

3.4.1 State-Space

A “state-space” is a mathematical model of a physical system that is a set of inputs, outputs, and state variables related to first-order differential equations. Simscape™ offers the ability to generate a general linearized “state-space” with specified p inputs, q outputs and n state variables written in the following continuous time-invariant form:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}
\] (26)

where \( \dot{x}(t) \) is the “state vector”, \( y(t) \) is the “output vector”, \( u(t) \) is the “input vector”, \( A \) is the “state matrix”, \( B \) is the “input matrix”, \( C \) is the “output matrix”, and \( D \) is “feedthrough matrix”. For example, an “input perturbation” in the form of a “chirp” signal acts as an input and the “output perturbation” is the electrical impedance measured by a voltage sensor. Subsequent to linearization, Simscape™ outputs a “state-space” containing the matrices \( A, B, C, D \) that can be exported and transformed into an “explicit” discrete time-invariant form, whereby magnitude, phase, and frequency information can be obtained in the following way in MATLAB:

```matlab
syne = ss(syad, 'explicit');
magnitude, phase, frequency = bode(syne);
```

where \( \text{linsys} \) is a “state-space” structure that contains the \( A, B, C, D \) matrices that can accessed through indexing (i.e. \( \text{linsys.A} \)).

3.4.2 Electrical Impedance

Much of the pertinent information about the performance of a loudspeaker configuration can be obtained from an electrical impedance measurement, wherein the “voltage-divider” principle may be implemented to generate an electrical impedance curve by using a “sensing” resistor of a known value and measuring the voltage drop across it.

This method will be explained in section §4.1 and can be referenced from a plethora of scholarly and online sources. Determining the electrical impedance of the system is more straightforward in Simscape™ as a “voltage sensor” may be connected across a component in the electrical domain and output to a Bode plot upon linearization of the “state-space” of a multi-domain model. This can be seen in any of the complete Simscape™ multi-domain block diagrams available in the Appendix. Comparison of Simscape™ electrical impedance magnitude will be compared with actual electrical impedance measurement data in sections §4.1.1, §4.1.2, and §4.1.3 as well as an explanation of the loudspeaker performance information that can be obtained from an electrical impedance measurement.

3.4.3 Sound Pressure Level

Of interest to the loudspeaker designer is the on-axis sound pressure level, which is measured at a standard distance of 1 m (\( r = 1 \text{ m} \)). For a circular plane piston with time-harmonic oscillation in an infinite baffle (“half-space”), the general pressure expression is \( p(r) = \frac{\omega \rho}{2\pi r} U e^{jkr} \), where \( \rho \) is the density of air (kg/m³), \( c \) is the speed of sound in air (m/s), \( \omega \) is the radial frequency of the applied excitation signal (rad/s), \( r \) is the radius of evaluation (m), \( U \) is the volume velocity of the piston (m³/s). To obtain the value for volume velocity \( U \) needed to output pressure as a function of frequency, a volume-velocity sensor that is analogous to a current sensor in the electrical domain may be used; however, additional considerations must be implemented to consider the \( j \omega \) portion of the general pressure expression. The relationship between pressure \( p \) and volume velocity \( U \) for a linear inerterance \( L \) (kg/m⁴) is \( p = L \frac{\partial U}{\partial t} \), where \( \frac{\partial U}{\partial t} = j\omega U \); therefore, integration of the signal received by the volume velocity sensor over an inerterance \( L \) of arbitrary value will result in an expression that can be multiplied by a constant whose value is equal to \( \frac{2\pi}{\omega} \) to obtain a curve of sound pressure level versus frequency. This method of determining the general pressure expression can be seen in any of the complete Simscape™ multi-domain block diagrams available in the Appendix.

![Figure 17](image1.png)  
**Figure 17** Electrical impedance magnitude for each Simscape™ multi-domain model in the frequency range of interest.

![Figure 18](image2.png)  
**Figure 18** Sound pressure level for each Simscape™ multi-domain model in the frequency range of interest.
It can be seen that the spherical enclosure offers a smoothly varying sound pressure level response for the Active Radiator and Active & Passive Radiator loudspeaker configurations; however, for Active Radiator and Port configuration, the sound pressure level response has a sharper drop-off at low frequencies with the second resonance frequency of the port being noticeably more prominent. Comparison of Simscape™ sound pressure level will be compared with actual sound pressure level measurement data in §4.4.

4 Experimental Measurement & Analysis

4.1 Electrical Impedance Measurement & Analysis

Much of the pertinent information about loudspeaker performance can be obtained from the measurement of the total electrical impedance of a loudspeaker system, wherein the proven “constant voltage” method may be implemented to obtain a measure of impedance versus frequency for the active radiator of interest. Fig. 19 shows the experimental setup to obtain the total electrical impedance curve of the isolated active radiator as well as for each loudspeaker configuration. For all electrical impedance measurements, an NI DAQ USB-4431 (see Fig. 19) coupled with a MATLAB script provided by a colleague (Andrew Doyle) was used for real-time post-processing and data acquisition. Practical implementation of the “constant voltage” method requires an output channel across the terminals of the active radiator as well as two input channels measuring the voltage, one of which measures the voltage “drop” across the active radiator (V_{Speaker}) and the other measuring the voltage “drop” across a “sensing” resistor (V_{Sense}). The rating of the “sensing” resistor (R_{Sense}) is arbitrary but should have a sensible precision (1 %) and a sufficient power rating. A “sensing” resistor rated at 50-ohm was chosen for all total electrical impedance measurements. The total electrical impedance can therefore be achieved by the following relationships:

\[
\bar{Z} = \frac{V_{\text{Speaker}}}{\bar{i}} \text{[ohm]} \tag{27}
\]

\[
Z = \left(\frac{\bar{V}_{\text{Speaker}}}{\bar{V}_{\text{Sense}}}\right) R_{\text{Sense}} \text{[ohm]} \tag{28}
\]

where the tilde (~) above the quantities of \(\bar{i}, \bar{V}\), and \(\bar{Z}\) denotes that the quantity is a complex phasor with both magnitude and phase components.

From inspection of the impedance magnitude versus frequency curve in Fig. 20 it can be seen that the multi-domain Simscape™ model offers a reasonable prediction for system performance. Prior to the implementation of all multi-domain Simscape models, a vacuum chamber impedance measurement was performed on the active radiator to assess the accuracy of the Thiele-Small parameters provided on specification sheet for the Tang Band® W3-2088S0F “micro” subwoofer. From this measurement it was determined that the approximate free-air resonance frequency (f_{SA}) of the active radiator was 59.3 Hz, a percentage difference of approximately 27.4%; therefore, it was deemed necessary to perform and “added-mass” measurement to determine better approximations of the suspension stiffness (k_{MS}) and effective diaphragm mass (M_{MS}) as well as reiterate the linear analysis of the Simscape™ model to obtain a suitable representation of the measured impedance response. The values in Table 4 in §3 represent the findings of this process and were subsequently applied to all Simscape™ models. The deviation in the impedance magnitude response above approximately 160 Hz is due to the fact that the voice coil inductance (L_{VC}) varies with frequency and diaphragm displacement and this is not compensated for in the Simscape™ model.

4.1.1 Active Radiator in a Spherical Baffle

From inspection of the impedance magnitude versus frequency curve in Fig. 21 it can be seen that the multi-domain Simscape™ model offers a reasonable prediction for system performance; however, the presence of the passive radiator resonance on the impedance magnitude curve for Simscape™ model is not present. Further measurement of the passive radiator mechanical characteristics would have proven valuable for obtaining better approximations of pertinent Thiele-Small parameters to alleviate this discrepancy; however, due to time constraints this initiative was not undertaken.

![Figure 19 Measurement apparatus for the determination of total electrical impedance of the Tang Band® W3-2088S0F and all loudspeaker configurations.](image-url)

![Figure 20 Comparison on impedance magnitude versus frequency for experimental measurements and the associated Simscape™ model. Note: [Fill] denotes a measurement taken with the acoustic polyfill added.](image-url)
measurements were conducted in a semi-
undertaken in a semi-
char
spherical cap
4.2
seen that the model incorrectly represents this resonance.
which may be the
resonance at about twice the frequency of the system resonance,
in
the Configuration 3 [No Fill] (Measured) impedance magnitude
port, or not a
compressibility, assuming laminar flow along the length of the
approximating the radiation impedance of the port termination
This discrepancy could have arisen for several reasons:
denotes a measurement
taken with the acoustic polyfill added.

4.1.3 Active Radiator and Port in a Spherical Baffle

From inspection of the impedance magnitude versus frequency curve in Fig. 22 it can be seen that the multi-domain Simscape™ model offers a somewhat imprecise prediction for system performance, most notably there is a discrepancy between the impedance magnitudes at system resonance.

This discrepancy could have arisen for several reasons: approximating the radiation impedance of the port termination as a piston in an infinite baffle and therefore disregarding compressibility, assuming laminar flow along the length of the port, or not accounting for other pertinent loss mechanisms. For the Configuration 3 [No Fill] (Measured) impedance magnitude in Fig. 22, it can also be seen that there is a small, broad resonance at about twice the frequency of the system resonance, which may be the result of standing waves in the port. It can be seen that the model incorrectly represents this resonance.

4.2 Directivity Measurements

As verification of the theoretical formulation for a spherical cap set in a sphere, a measure of the directivity characteristics for the base loudspeaker configuration was undertaken in a semi-anechoic environment. The directivity measurements were conducted in a semi-anechoic chamber that is part of the AURAS (AUralization and Reproduction of Acoustic Sound fields) facility that is a part of the research conducted by members of SPRAL (Sound Perception and Room Acoustics Laboratory). The dimensions of the semi-anechoic chamber are 11’ by 14’ by 7’ with an effective anechoic environment down to approximately 200 Hz due to the length of the wedges employed over the surfaces of the room. A separation of approximately 3 meters between the microphone and the acoustic center of the loudspeaker was achieved, allowing for “far-field” measurements for frequencies above approximately 115 Hz. A mechanically-driven turntable apparatus upon which the loudspeaker was mounted was utilized, with markings at 10-degree intervals from 0 to 180 degrees. A revolution of 180 degrees was deemed sufficient as the loudspeaker exhibits symmetry on the axis of revolution. A B&K ½ “free-field” microphone with a frequency response of 3.75 to 20000 Hz (±2dB) was utilized to take pressure measurements for the subsequent generation of normalized directivity plots.

An Audeze Deckard headphone amplifier was used for test signal output and loudspeaker amplification and a UAD Apollo Twin mkII was used as a preamplifier and DAC for pressure signal input. The test signal was a logarithmic sine sweep (i.e. “chirp”) from 20 Hz to 2000 Hz with each “chirp” being 0.5 seconds long and having a Hann window applied to beginning and end of each “chirp”. 20 averages were used, resulting in a total test signal length of 10 seconds for each 10-degree interval of revolution. MATLAB was used for signal generation and real-time post-processing on input raw pressure data. A preliminary measurement of the ambient noise conditions in the semi-anechoic chamber revealed that environmental noise may affect the directivity characteristics for the first two third-octave frequency bands of interest (31.5 Hz and 62 Hz), for which all methods by which the environmental noise could be reduced through the implementation of digital filters, averaging, and test signal choice was considered. Fig. 23 shows the extent of the environmental noise compared to the sound pressure level measured at all 10-degree intervals of revolution.
Inspection of the normalized directivity plots shows reasonable agreement with the results predicted by the theoretical formulation; however, there are some deficiencies that are worth discussing. Most notably is the difference between the measured and theoretical results for directivity at 500 Hz, for which there are prominent nodes at 90 degrees. In §2 it was shown that the construction of the spherical enclosure was the result of the combination of two hemispherical sections with D-profile weatherstrip used as a pressure seal between the two sections, as well as wing turn latches for ease of accessibility and for providing a further impediment against pressure leaks. It appears that at 500 Hz, either the properties of the D-profile weatherstrip material inhibited acoustic radiation, structural resonances of the measurement apparatus were excited, or there was a leak in pressure, resulting in a normalized directivity pattern resembling somewhat of an acoustic dipole response.

The deviation between the measured and theoretical results for directivities at 125 Hz and 250 Hz from 90 to 180 degrees may reasonably be assumed to be due to the use of a rigid flat cap (identifier [C] in Table 4) on the “back-side” of the loudspeaker, thereby changing the boundary conditions put forth in the theoretical formulation. Lastly, the deviation between the measured and theoretical results for directivities at 31.5 Hz and 62 Hz is to be expected due to the excessive amounts of environmental noise, as can be seen in Fig. 23; however, the directivities for each of these frequencies is fairly omnidirectional, as is to be expected. Overall, the directivities for each of the third-octave band frequencies of interest agree moderately well with those put forth by the theoretical formulation, especially given the experimental circumstances. Directivity measurements were also conducted for each of the other loudspeaker configurations, those employing a passive radiator or a port. It was noted that the normalized directivity patterns for each of these configurations did not change considerably and are therefore are not displayed. Further verification of the acoustic characteristics of a spherical cap set in a sphere could be obtained through the use of an intensity probe comprised of two closely spaced pressure microphones, wherein the use such microphones allows for the approximation of a particle velocity that can be used to calculate on-axis acoustic intensity and power.

4.4 Sound Pressure Level Measurements

It was mentioned in the §0 that a spherical enclosure ideally offers the flattest frequency response with an average deviation in sound pressure level of ±0.5 dB in sound pressure level; however, it can be seen in Fig. 28 that the “on-axis” (0 degrees) sound pressure level for all three loudspeaker configurations varies considerably, with the largest deviation being approximately 50 dB in the frequency range above where the environmental noise is not impinging on the measurement. The most notable deviation is the dip in sound pressure level at frequency at about 110 Hz. This dip in response is not present on the sound pressure level versus frequency response plot provided on the specification sheet for the Tang Band® W3-208850F and is decidedly not the result of the implementation of the passive radiator and port as the dip is present for all three loudspeaker configurations. It is my assumption that either the structural resonances of the measurement apparatus or standing waves within the spherical enclosure are suppressing sound.

**Figure 24** A measure of sound pressure level at 10-degree intervals versus frequency (250 Hz to 2000 Hz). Note. The vertical black dotted lines denote third-octave frequency bands. Note. Logarithmic x-axis.

**Figure 25** Normalized directivity plots for 31.5 Hz (left) and 62 Hz (right). Note. The solid line denotes the theoretical directivity and the dotted line denotes the measured directivity.

**Figure 26** Normalized directivity plots for 125 Hz (left) and 250 Hz (right). Note. The solid line denotes the theoretical directivity and the dotted line denotes the measured directivity.

**Figure 27** Normalized directivity plots for 500 Hz (left) and 1000 Hz (right). Note. The solid line denotes the theoretical directivity and the dotted line denotes the measured directivity.

**Figure 28** The deviation between the measured and theoretical results for directivities at 125 Hz and 250 Hz from 90 to 180 degrees may reasonably be assumed to be due to the use of a rigid flat cap (identifier [C] in Table 4) on the “back-side” of the loudspeaker, thereby changing the boundary conditions put forth in the theoretical formulation. Lastly, the deviation between the measured and theoretical results for directivities at 31.5 Hz and 62 Hz is to be expected due to the excessive amounts of environmental noise, as can be seen in Fig. 23; however, the directivities for each of these frequencies is fairly omnidirectional, as is to be expected. Overall, the directivities for each of the third-octave band frequencies of interest agree moderately well with those put forth by the theoretical formulation, especially given the experimental circumstances. Directivity measurements were also conducted for each of the other loudspeaker configurations, those employing a passive radiator or a port. It was noted that the normalized directivity patterns for each of these configurations did not change considerably and are therefore are not displayed. Further verification of the acoustic characteristics of a spherical cap set in a sphere could be obtained through the use of an intensity probe comprised of two closely spaced pressure microphones, wherein the use such microphones allows for the approximation of a particle velocity that can be used to calculate on-axis acoustic intensity and power.

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radiation around this frequency. Even though the entire inside of the enclosure is filled with acoustic polyfill to suppress standing waves, standing waves still have the probability of being present and a further investigation into this phenomenon would be needed for clarification.

The utilization of the port for the third loudspeaker configuration can be seen as having the benefit of increasing the low frequency sound pressure level output around the resonance frequency of the system with an increase of approximately 6 dB. The benefit of utilizing the passive radiator for the second loudspeaker configuration is unclear as the presence of low frequency environmental noise within the vicinity of the passive radiator’s resonance frequency obscures a quantifiable assessment of its contributable performance; however, there is a slight boost in sound pressure level above 500 Hz which could be attributable to the active and passive radiator being in-phase at higher frequencies. Overall, the sound pressure level is quite erratic over the frequency range of interest, suggesting that all three loudspeaker configurations would be better suited for use a subwoofer through the use of low-pass filtering on the input signal. It was stated that to obtain the theoretical calculation of sound pressure level in §1.2 a uniform piston velocity \( u_0 \) was needed; however, results infer that a singular value does not exist for all the frequencies of interest and would further require an accelerometer measurement of the active radiator at each individual frequency of interest.

5 Discussion

Given the innate complexity of even a simplistic loudspeaker system such as the three loudspeaker configurations analyzed in this study, one could conduct and present an entire expounded section on the non-linearities present in each lumped-element component used in the Simscape™ multi-domain models (see Appendix). Such was the intent to include these non-linearities, as Klippel GmbH system diagnostic information was obtained from Vance Dickason for the Tang Band® W3-2088S0F; however, due to time constraints, this initiative was not undertaken. The utilization of COMSOL Multiphysics® for the analysis of sound-structure interaction for each of the loudspeaker configurations was also considered as the CAD software Autodesk® Fusion 360 allows for the generation of representative 3-dimensional meshes that can be imported into COMSOL Multiphysics® for analysis. This analysis would have been advantageous in predicting the deviation in the measured directivity for the 500 Hz third-octave band and detecting other structural resonance that would inhibit acoustic radiation, as well as for predicting spherical harmonic standing waves within the spherical enclosure. In §2.4 it was noted that the proper “alignments” for the port were not undertaken and that the reason was due to dimensional constraints imposed by both the spherical enclosure and the 3D printers available at the MakerBot™ Innovation Center. If the resolution and dimensional constraints of the 3D printers were not a factor, one may construct a port that maximizes the inner 3-dimensional space of the spherical enclosure by “snaking” the port in a spiral configuration to add additional length for proper “alignment”; however, a port configuration of this nature might require a transmission line network for representation in a multi-domain model or an FEA approach in COMSOL Multiphysics®.

Acknowledgements

I would like to thank Dr. Stephen Thompson for his knowledge, support, and constructive criticism throughout the development of this study.

I would like to thank my colleagues Andrew Doyle, Lane Miller, and Molly Smallcomb for assisting in the setup and execution of various experimental setups throughout the development of this study.

References


*References [7] and [8] were utilized as a knowledge base for the presentation of concepts and are not referenced above in the body of the paper.
Appendix

Table A.1: Phase Angle Coefficients

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Table A.2: Amplitude Coefficients

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Figure A.1 Simscape™ (MathWorks®) multi-domain model for the first loudspeaker configuration (see. Fig. 7 for physical representation)

Figure A.2 Simscape™ (MathWorks®) multi-domain model for the second loudspeaker configuration (see. Fig. 8 for physical representation)

Figure A.3 Simscape™ (MathWorks®) multi-domain model for the third loudspeaker configuration (see. Fig. 9 for physical representation)